Chapter 5
Risk-Seeking Agent

In previous section we represented agent's perceived risk by a measure that reflects the dispersion of his revenue stream. Although the dispersion of possible outcomes has been widely used as the measure of risk (Pratt 1964; Rothschild and Stiglitz 1970; Stiglitz 1974; Levy 1992; Fukunaga and Huffman 2009; Lewis and Bajari 2014) it fails to capture observable behavior in risky settings. In this section we extend our principal-agent analysis to risk-seeking agent. We note that there is an ongoing evaluation of risk attitudes in an attempt to explain peoples' behavior when faced with risky choices. For instance Prospect Theory claims to offer a better model that covers discrepancies observed elsewhere (Kahneman and Tversky 1979; Tversky and Kahneman 1992). Prospect Theory claims that people are less sensitive to the variation of the probability of outcomes compared to the expectation, and losses loom larger than gains. Furthermore, empirical evidences indicates that decision makers prefer expressions of risk in terms of the expected value at stake, and they appear to be risk-averse when dealing with a risky alternative whose possible outcomes are generally good and tend to be risk-seeking when dealing with a risky alternative whose possible outcomes are generally poor (March and Shapira 1987; Filiz-Ozbay et al. 2013).

In our principal-agent setting with a risk-seeking agent we propose that an agent perceives a greater loss when he is charged a larger penalty rate for each unit of downtime and also when the probability of being in the failed state goes up. The agent's penalty rate at any point of time can be modeled as pB where B is a Bernoulli random variable that takes value 0 with probability $P(0) = \mu/(\lambda + \mu)$ and value 1 with probability $P(1) = \lambda/(\lambda + \mu)$. For simplicity denote momentarily $a \equiv P(1)$. In this section we adopt the following risk measure:

$$r(a) \equiv p\left(a - \frac{1}{2}\right)_+ \quad \text{for } a \in [0, 1]$$

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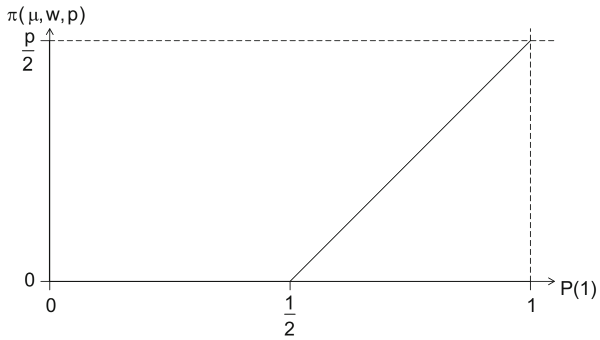


Fig. 5.1 $\pi(\mu, w, p)$ as a function of $P(1)$ when $\eta = -1$

We note that $R(pB) \equiv r(a)$ satisfies the properties of *monotonicity*, *sub-additivity* and *positive homogeneity* of a coherent risk measure but fails to satisfy the property of *translation invariance*, since $R(pB)$ is independent of the expectation of pB (Artzner et al. 1999).

Risk premium of a risk-seeking agent is the $ value considered by the agent as extra gains to his revenue stream. As a consequence, just for the risk-seeking agent we modify the risk premium defined earlier in (4.1), in a manner that reflects the expected amount at stake instead of the dispersion of the revenue stream:

$$\pi(\mu, w, p) \equiv -\eta p \left(P(1) - \frac{1}{2}\right)_+ = -\eta p \left(\frac{\lambda}{\lambda + \mu} - \frac{1}{2}\right)_+ \tag{5.1}$$

Note that for risk-seeking agent $\eta < 0 \Rightarrow \pi(\mu, w, p) \geq 0$, and adding such a risk premium to a risk-neutral agent's expected utility rate (as in (5.2)) implies risk-seeking. Figure 5.1 depicts $\pi(\mu, w, p)$ as a function of $P(1)$ for $\eta = -1$.

The representation of the risk premium in (5.1) is consistent with the properties of risk in the Prospect Theory (Kahneman and Tversky 1979; Tversky and Kahneman 1992) and the empirical findings in (March and Shapira 1987): The risk premium is zero when $P(1)$ is lower than $1/2$. The risk premium increases with $P(1)$ linearly when $P(1)$ exceeds one half, and reaches its peak when $P(1) = 1$.

Denote $\overline{\eta} \equiv -\eta > 0$. Modifying (3.2), the risk-seeking agent's expected utility rate is:

$$u_A(\mu; w, p) = \left(w - \frac{p\lambda}{\lambda + \mu} - \mu + \overline{\eta} p \left(\frac{\lambda}{\lambda + \mu} - \frac{1}{2}\right)_+\right)_+ \quad \text{for } w > 0, p > 0, \mu \geq 0 \tag{5.2}$$

Since the analysis is different for $\overline{\eta} \in (0, 8/9)$, $\overline{\eta} \in [8/9, 2)$ and $\overline{\eta} \geq 2$, therefore, when $\overline{\eta} \in (0, 8/9)$ we consider the agent as *weakly risk-seeking*, when $\overline{\eta} \in [8/9, 2)$ we consider the agent as *moderately risk-seeking*, and when $\overline{\eta} \geq 2$ we consider the agent as *strongly risk-seeking*. We assume, say for historic reasons, that both the agent and the principal know not only the agent's type as risk-seeking but also the value of $\overline{\eta}$.

The expression for the principal's expected revenue rate $\Pi_P(w, p; \mu)$ remains the same as (3.3).

Before examining the details of the optimal contracts we discuss a potential case of the agent compensating the principal at times during the contract. Such occurrence of utility transfer from an risk-seeking agent to the principal can have one of two forms: either the compensation rate is non-positive ($w \leq 0$), or the principal is guaranteed a positive expected revenue rate even with her unit in the failed state forever ($-w + p > 0$ if $\mu = 0$). Under our setting of undetermined contract horizon it is unrealistic to accept that the agent might compensate the principal when the unit is forever in the failed state. Therefore the occurrence of a non-positive compensation rate ($w \leq 0$) has be ruled out in the definition of the Strategy Set (Definition 2.1). Nevertheless, the possibility of the principal receiving a positive expected revenue rate with a failed unit has to be considered. Therefore we extend the definition of the *Set of Admissible Solutions* (Definition 2.3) as follows.

Definition 5.1 (*Set of Admissible Solutions*). *The* ***set of admissible solutions*** *for the principal-agent problem* $\mathfrak{P}$ *is the set* $\mathfrak{s}(\mathfrak{P})$ *of all strategies* $((w, p), \mu) \in \mathfrak{S}(\mathfrak{P})$ *for which:*

(a) $\nexists$ $((w', p'), \mu') \in \mathfrak{S}(\mathfrak{P})$ *such that* $((w', p'), \mu') \succeq ((w, p), \mu)$ *– there is no other strategy that weakly dominates* $((w, p), \mu)$.
(b) $\Pi_P(w, p; \mu) > \underline{\Pi_P}$ *and* $u_A(\mu; w, p) \geq \underline{u_A}$.
(c) *If* $\mu = 0$, *then* $w \geq p$.

We denote the part inside the brackets in Eq. (5.2) as

$$u(\mu) \equiv \begin{cases} w - \dfrac{\overline{\eta}p}{2} - \dfrac{(1-\overline{\eta})p\lambda}{\lambda + \mu} - \mu, \; \mu \in [0, \lambda] \\ w - \dfrac{p\lambda}{\lambda + \mu} - \mu, \; \mu > \lambda \end{cases} \tag{5.3}$$

Note that $u(\mu)$ is differentiable everywhere for $\mu \geq 0$ except at $\mu = \lambda$. When $\mu \in [0, \lambda)$:

$$\frac{du(\mu)}{d\mu} = \frac{(1-\overline{\eta})p\lambda}{(\lambda+\mu)^2} - 1, \; \lim_{\mu \to 0^+} \frac{du(\mu)}{d\mu} = \frac{1-\overline{\eta}}{\lambda}\left(p - \frac{\lambda}{1-\overline{\eta}}\right)$$

$$\lim_{\mu \to \lambda^-} \frac{du(\mu)}{d\mu} = \frac{1-\overline{\eta}}{4\lambda}\left(p - \frac{4\lambda}{1-\overline{\eta}}\right), \; \frac{d^2u(\mu)}{d\mu^2} = -\frac{2(1-\overline{\eta})p\lambda}{(\lambda+\mu)^3}$$

and when $\mu > \lambda$:

$$\frac{du(\mu)}{d\mu} = \frac{p\lambda}{(\lambda+\mu)^2} - 1, \lim_{\mu\to\lambda+} \frac{du(\mu)}{d\mu} = \frac{p-4\lambda}{4\lambda}$$

$$\lim_{\mu\to+\infty} \frac{du(\mu)}{d\mu} = -1 < 0 \text{ and } \frac{d^2u(\mu)}{d\mu^2} = -\frac{2p\lambda}{(\lambda+\mu)^3} < 0$$

The positivity or negativity of the above derivatives indicate the direction of monotonicity and the concavity/convexity of the function $u(\mu)$ over $[0,\lambda)$ and $(\lambda,+\infty)$. Table 5.1 summarizes these indicators for various regions of the space $\mathbb{R}^2_+$ for pairs of $(\overline{\eta}, p)$. In the table $u_\mu(\cdot) = \lim_{\mu\to\cdot} du/d\mu$, and $u_\mu(\cdot^+)$ represents the limit of $u_\mu(\mu)$ as μ approaches $(\cdot)$ from above, and similarly $u_\mu(\cdot^-)$ represents the limit of $u_\mu(\mu)$ as μ approaches $(\cdot)$ from below.

5.1 Optimal Strategies for the Weakly Risk-Seeking Agent

Note that agent's expected utility rate (see (5.2)) increases and principal's expected profit rate (see (3.3)) decreases in w, therefore for any value of p the principal can maximize her expected profit rate by lowering w yet safeguarding agent's participation by setting the agent's expected utility rate equal to his reservation utility rate. Although the principal cannot contract directly on the agent's service capacity, she anticipates the agent optimizing his expected utility rate when offered a contract. That is, for any w and p values proposed by the principal, the agent computes the μ that maximizes his expected utility rate and subsequently decides whether to accept the contract or not, by solving the following optimization problem:

$$\max_{\mu\geq 0} u(\mu) = \max_{\mu\geq 0} \left\{ w - \frac{p\lambda}{\lambda+\mu} - \mu + \overline{\eta}p\left(\frac{\lambda}{\lambda+\mu} - \frac{1}{2}\right)_+ \right\} \quad (5.4)$$

The agent's optimal service capacity is denoted by $\mu^*(w,p) = \text{argmax}_{\mu\geq 0}\, u(\mu)$.

Before proceeding to derive the agent's optimal strategy we introduce some notation:

$$\overline{p}_1 \equiv \frac{\lambda}{1-\overline{\eta}}, \quad \overline{p}_2 \equiv \frac{16\left(2-\overline{\eta}-2\sqrt{1-\overline{\eta}}\right)\lambda}{\overline{\eta}^2} \quad (5.5)$$

and the following identity is verified using the definition of $\overline{p}_2$:

$$\overline{w}_2 \equiv \frac{\overline{\eta}\overline{p}_2}{2} + 2\sqrt{(1-\overline{\eta})\overline{p}_2\lambda} - \lambda = 2\sqrt{\overline{p}_2\lambda} - \lambda \quad (5.6)$$

Note that $\overline{p}_1, \overline{p}_2$ and $\overline{w}_2$ are functions of λ and $\overline{\eta}$. However we suppress $(\lambda, \overline{\eta})$.

Table 5.1 Indicators of the monotonicity and the concavity/convexity of function $u(\mu)$ in (5.3)

Case		$u_\mu(0^+)$	Over $[0,\lambda)$ $u(\mu)$ is	$u_\mu(\lambda^-)$	$u_\mu(\lambda^+)$	Over $(\lambda,+\infty)$ $u(\mu)$ is	$u_\mu(+\infty)$
$\overline{\eta} \in \left(0, \frac{3}{4}\right]$	$p \in \left(0, \frac{\lambda}{1-\overline{\eta}}\right]$	≤ 0	Concave	< 0	< 0	Concave	< 0
	$p \in \left(\frac{\lambda}{1-\overline{\eta}}, 4\lambda\right]$[a]	> 0	Concave	< 0	≤ 0	Concave	< 0
	$p \in \left(4\lambda, \frac{4\lambda}{1-\overline{\eta}}\right]$[a]	> 0	Concave	≤ 0	> 0	Concave	< 0
	$p \in \left(\frac{4\lambda}{1-\overline{\eta}}, +\infty\right)$	> 0	Concave	> 0	> 0	Concave	< 0
$\overline{\eta} \in \left(\frac{3}{4}, 1\right)$	$p \in (0, 4\lambda]$	< 0	Concave	< 0	≤ 0	Concave	< 0
	$p \in \left(4\lambda, \frac{\lambda}{1-\overline{\eta}}\right]$[b]	≤ 0	Concave	< 0	> 0	Concave	< 0
	$p \in \left(\frac{\lambda}{1-\overline{\eta}}, \frac{4\lambda}{1-\overline{\eta}}\right]$[b]	> 0	Concave	≤ 0	> 0	Concave	< 0
	$p \in \left(\frac{4\lambda}{1-\overline{\eta}}, +\infty\right)$	> 0	Concave	> 0	> 0	Concave	< 0
$\overline{\eta} \in [1, +\infty)$	$p \in (0, 4\lambda]$[c]	< 0	Convex	< 0	≤ 0	Concave	< 0
	$p \in (4\lambda, +\infty)$	< 0	Convex	< 0	> 0	Concave	< 0

[a] Note that $\overline{\eta} \in (0, 3/4] \Rightarrow 4\lambda/(1-\overline{\eta}) > 4\lambda \geq \lambda/(1-\overline{\eta})$
[b] Note that $\overline{\eta} \in (3/4, 1) \Rightarrow 4\lambda/(1-\overline{\eta}) > \lambda/(1-\overline{\eta}) > 4\lambda$
[c] Note that $\overline{\eta} > 1 \Rightarrow 4\lambda > 0 > \lambda/(1-\overline{\eta}) > 4\lambda/(1-\overline{\eta})$

Next we introduce a number of technical lemmas (see proofs in the Appendix).

Lemma 5.2. *Let* $1 > \overline{\eta} > 0$ *and* $\lambda > 0$.

(a) *If* $\dfrac{4\left(\sqrt{1-\overline{\eta}/2}-\sqrt{1-\overline{\eta}}\right)^2\lambda}{\overline{\eta}^2} > p > 0$, *then* $0 > \dfrac{\overline{\eta}p}{2} + 2\sqrt{(1-\overline{\eta})p\lambda} - \lambda$.

(b) *If* $p > \dfrac{4\left(\sqrt{1-\overline{\eta}/2}-\sqrt{1-\overline{\eta}}\right)^2\lambda}{\overline{\eta}^2}$, *then* $\dfrac{\overline{\eta}p}{2} + 2\sqrt{(1-\overline{\eta})p\lambda} - \lambda > 0$.

(c) *If* $p = \dfrac{4\left(\sqrt{1-\overline{\eta}/2}-\sqrt{1-\overline{\eta}}\right)^2\lambda}{\overline{\eta}^2}$, *then* $\dfrac{\overline{\eta}p}{2} + 2\sqrt{(1-\overline{\eta})p\lambda} - \lambda = 0$.

Lemma 5.3. *Let* $1 > \overline{\eta} > 0$ *and* $\lambda > 0$, *then* $\lambda/(1-\overline{\eta}) > 4\left(\sqrt{1-\overline{\eta}/2}-\sqrt{1-\overline{\eta}}\right)^2 \lambda/\overline{\eta}^2$.

Lemma 5.4. *Let* $2 > \overline{\eta} > 0$ *and* $\lambda > 0$.

(a) *If* $\dfrac{2\lambda}{2+\overline{\eta}-2\sqrt{2\overline{\eta}}} > p > \dfrac{2\lambda}{2+\overline{\eta}+2\sqrt{2\overline{\eta}}}$, *then* $0 > \left(1-\dfrac{\overline{\eta}}{2}\right)p - 2\sqrt{p\lambda} + \lambda$.

(b) *If* $\dfrac{2\lambda}{2+\overline{\eta}+2\sqrt{2\overline{\eta}}} > p > 0$ *or* $p > \dfrac{2\lambda}{2+\overline{\eta}-2\sqrt{2\overline{\eta}}}$, *then* $\left(1-\dfrac{\overline{\eta}}{2}\right)p - 2\sqrt{p\lambda} + \lambda > 0$.

(c) *If* $p = \dfrac{2\lambda}{2+\overline{\eta}+2\sqrt{2\overline{\eta}}}$ *or* $p = \dfrac{2\lambda}{2+\overline{\eta}-2\sqrt{2\overline{\eta}}}$, *then* $\left(1-\dfrac{\overline{\eta}}{2}\right)p - 2\sqrt{p\lambda} + \lambda = 0$.

Lemma 5.5. *Let* $\overline{\eta} > 0$ *and* $\lambda > 0$, *then* $4\lambda > 2\lambda/\left(2+\overline{\eta}+2\sqrt{2\overline{\eta}}\right)$.

Lemma 5.6. *Let* $\lambda > 0$.

(a) *If* $\dfrac{8}{9} > \overline{\eta} > 0$, *then* $\dfrac{2\lambda}{2+\overline{\eta}-2\sqrt{2\overline{\eta}}} > \dfrac{\lambda}{1-\overline{\eta}}$.

(b) *If* $1 > \overline{\eta} > \dfrac{8}{9}$, *then* $\dfrac{\lambda}{1-\overline{\eta}} > \dfrac{2\lambda}{2+\overline{\eta}-2\sqrt{2\overline{\eta}}}$.

(c) *If* $\overline{\eta} = \dfrac{8}{9}$, *then* $\dfrac{2\lambda}{2+\overline{\eta}-2\sqrt{2\overline{\eta}}} = \dfrac{\lambda}{1-\overline{\eta}}$.

Lemma 5.7. *Let* $1 > \overline{\eta} > 0$ *and* $\lambda > 0$.

(a) *If* $\dfrac{16\left(2-\overline{\eta}-2\sqrt{1-\overline{\eta}}\right)\lambda}{\overline{\eta}^2} > p > 0$, *then* $0 > \dfrac{\overline{\eta}p}{2} - 2\left(1-\sqrt{1-\overline{\eta}}\right)\sqrt{p\lambda}$.

(b) *If* $p > \dfrac{16\left(2-\overline{\eta}-2\sqrt{1-\overline{\eta}}\right)\lambda}{\overline{\eta}^2}$, *then* $\dfrac{\overline{\eta}p}{2} - 2\left(1-\sqrt{1-\overline{\eta}}\right)\sqrt{p\lambda} > 0$.

(c) *If* $p = 0$ *or* $p = \dfrac{16\left(2-\overline{\eta}-2\sqrt{1-\overline{\eta}}\right)\lambda}{\overline{\eta}^2}$, *then* $\dfrac{\overline{\eta}p}{2} - 2\left(1-\sqrt{1-\overline{\eta}}\right)\sqrt{p\lambda} = 0$.

Lemma 5.8. *Let* $\lambda > 0$.

(a) *If* $\frac{8}{9} > \overline{\eta} > 0$, *then* $\frac{16\left(2-\overline{\eta}-2\sqrt{1-\overline{\eta}}\right)\lambda}{\overline{\eta}^2} > \frac{\lambda}{1-\overline{\eta}}$.

(b) *If* $1 > \overline{\eta} > \frac{8}{9}$, *then* $\frac{\lambda}{1-\overline{\eta}} > \frac{16\left(2-\overline{\eta}-2\sqrt{1-\overline{\eta}}\right)\lambda}{\overline{\eta}^2}$.

(c) *If* $\overline{\eta} = \frac{8}{9}$, *then* $\frac{16\left(2-\overline{\eta}-2\sqrt{1-\overline{\eta}}\right)\lambda}{\overline{\eta}^2} = \frac{\lambda}{1-\overline{\eta}}$.

Lemma 5.9. *Let* $1 > \overline{\eta} > 0$ *and* $\lambda > 0$, *then* $4\lambda/(1-\overline{\eta}) > 16\left(2-\overline{\eta}-2\sqrt{1-\overline{\eta}}\right)\lambda/\overline{\eta}^2 > 4\lambda$.

Lemmas 5.8 and 5.9 imply $\overline{\eta} \in (0, 3/4) \Rightarrow 4\overline{p}_1 > \overline{p}_2 > 4\lambda \geq \overline{p}_1 > 0$ and $\overline{\eta} \in (3/4, 8/9) \Rightarrow 4\overline{p}_1 > \overline{p}_2 > \overline{p}_1 > 4\lambda > 0$.

We present weakly risk-seeking agent's optimal response to any contract offers $(w, p) \in \mathbb{R}^2_+$ in Proposition 5.10.

Proposition 5.10. *Consider a weakly risk-seeking agent* ($\overline{\eta} \in (0, 8/9)$).

(a) *Given*

$$p \in (0, \overline{p}_1] \text{ and } w \geq \left(1 - \frac{\overline{\eta}}{2}\right) p \tag{5.7}$$

then the agent accepts the contract and installs $\mu^*(w, p) = 0$ *with resulting expected utility rate* $u_A(\mu^*(w, p); w, p) = w - (1 - \overline{\eta}/2)p \geq 0$. *The agent rejects the contract if both* $p \in (0, \overline{p}_1]$ *and* $w \in (0, (1 - \overline{\eta}/2)p)$.

(b) *Given*

$$p \in (\overline{p}_1, \overline{p}_2) \text{ and } w \geq \frac{\overline{\eta}p}{2} + 2\sqrt{(1-\overline{\eta})p\lambda} - \lambda \tag{5.8}$$

then the agent accepts the contract and installs $\mu^*(w, p) = \sqrt{(1-\overline{\eta})p\lambda} - \lambda$ *with resulting expected utility rate* $u_A(\mu^*(w, p); w, p) = w - \overline{\eta}p/2 - 2\sqrt{(1-\overline{\eta})p\lambda} + \lambda \geq 0$. *The agent rejects the contract if both* $p \in (\overline{p}_1, \overline{p}_2)$ *and* $w \in \left(0, \overline{\eta}p/2 + 2\sqrt{(1-\overline{\eta})p\lambda} - \lambda\right)$.

(c) *Given*

$$p = \overline{p}_2 \text{ and } w \geq \overline{w}_2 \tag{5.9}$$

then the agent accepts the contract and is indifferent about installing either $\mu^*(w, p) = \sqrt{(1-\overline{\eta})\overline{p}_2\lambda} - \lambda$ *or* $\mu^*(w, p) = \sqrt{\overline{p}_2\lambda} - \lambda$. *In both cases the agent receives* $u_A(\mu^*(w, p); w, p) = w - \overline{w}_2 \geq 0$. *If* $r \in (0, \overline{p}_2)$, *then there exists* w^* *such that* $\left((w^*, \overline{p}_2), \mu^* = \sqrt{(1-\overline{\eta})\overline{p}_2\lambda} - \lambda\right)$ *is the unique admissible solution (see Definition 5.1). If* $r = \overline{p}_2$, *there exists* w^* *such* *that* $\left((w^*, \overline{p}_2), \mu^* = \sqrt{\overline{p}_2\lambda} - \lambda\right)$ *and* $\left((w^*, \overline{p}_2), \mu^* = \sqrt{(1-\overline{\eta})\overline{p}_2\lambda} - \lambda\right)$ *are both admissible solutions. If* $r > \overline{p}_2$*, then there exists* w^* *such that* $\left((w^*, \overline{p}_2), \mu^* = \sqrt{\overline{p}_2\lambda} - \lambda\right)$ *is the unique admissible solution (for proof see Proposition 5.13). The agent rejects the contract if both* $p = \overline{p}_2$ *and* $w \in (0, \overline{w}_2)$*.*

(d) *Given*

$$p > \overline{p}_2 \text{ and } w \geq 2\sqrt{p\lambda} - \lambda \tag{5.10}$$

then the agent accepts the contract and installs $\mu^*(w,p) = \sqrt{p\lambda} - \lambda$ *with resulting expected utility rate* $u_A(\mu^*(w,p); w, p) = w - 2\sqrt{p\lambda} + \lambda \geq 0$*. The agent rejects the contract if both* $p > \overline{p}_2$ *and* $w \in \left(0, 2\sqrt{p\lambda} - \lambda\right)$*.*

Proof. According to Table 5.1, the behavior of $u(\mu)$ when $\overline{\eta} \in (0, 3/4]$ versus $\overline{\eta} \in (3/4, 8/9)$ is different. Therefore we prove the proposition separately for $\overline{\eta} \in (0, 3/4]$ and $\overline{\eta} \in (3/4, 8/9)$.

Case $\overline{\eta} \in (0, 3/4]$: According to Lemmas 5.8 part (a) and 5.9, $4\overline{p}_1 > \overline{p}_2 > 4\lambda \geq \overline{p}_1 > 0$. Figure 5.2 depicts the shape of $u(\mu)$ when $\overline{\eta} \in (0, 3/4]$ and the value of p falls in different ranges. The structure of the proof when $\overline{\eta} \in (0, 3/4]$ is depicted in Fig. 5.3.

Case $p \in \left(0, \overline{p}_1\right]$: According to Table 5.1, $u(\mu)$ is decreasing with respect to $\mu \geq 0$. Thus the agent's optimal service capacity is $\mu^*(w,p) = 0$ and from (5.3) $u(\mu^*(w,p)) = w - (1 - \overline{\eta}/2)p$. Note that $1 - \overline{\eta}/2 > 0$.

Subcase $w \in \left(0, (1 - \overline{\eta}/2)p\right)$: $u(\mu^*(w,p)) < 0$, therefore the agent rejects the contract.

Subcase $w \geq (1 - \overline{\eta}/2)p$: $u(\mu^*(w,p)) \geq 0$, thus the agent would accept the contract if offered.

Case $p \in \left(\overline{p}_1, 4\lambda\right]$: According to Table 5.1, the service capacity that maximizes $u(\mu)$ lies in $(0, \lambda)$. μ^* is computed from first order condition $du(\mu)/d\mu|_{\mu=\mu^*(w,p)} = 0 \Rightarrow \mu^*(w,p) = \sqrt{(1-\overline{\eta})p\lambda} - \lambda > 0$ and from (5.3) $u(\mu^*(w,p)) = w - \overline{\eta}p/2 - 2\sqrt{(1-\overline{\eta})p\lambda} + \lambda$. According to Lemmas 5.2 part (b) and 5.3, $\overline{\eta}p/2 + 2\sqrt{(1-\overline{\eta})p\lambda} - \lambda > 0$, therefore we examine the following subcases.

Subcase $w \in \left(0, \overline{\eta}p/2 + 2\sqrt{(1-\overline{\eta})p\lambda} - \lambda\right)$: $u(\mu^*(w,p)) < 0$, therefore the agent rejects the contract.

Subcase $w \geq \overline{\eta}p/2 + 2\sqrt{(1-\overline{\eta})p\lambda} - \lambda$: $u(\mu^*(w,p)) \geq 0$, thus the agent would accept the contract if offered.

Case $p \in \left(4\lambda, 4\overline{p}_1\right]$: According to Table 5.1, there is a service capacity that maximizes $u(\mu)$ for $\mu \in (0, \lambda]$ and a service capacity that maximizes $u(\mu)$ for $\mu > \lambda$. Denote the optimal service capacity in $(0, \lambda]$ by $\mu^*_{(0,\lambda]}(w,p)$. From the first order condition the optimal service capacity is $\mu^*_{(0,\lambda]}(w,p) = \sqrt{(1-\overline{\eta})p\lambda} - \lambda$

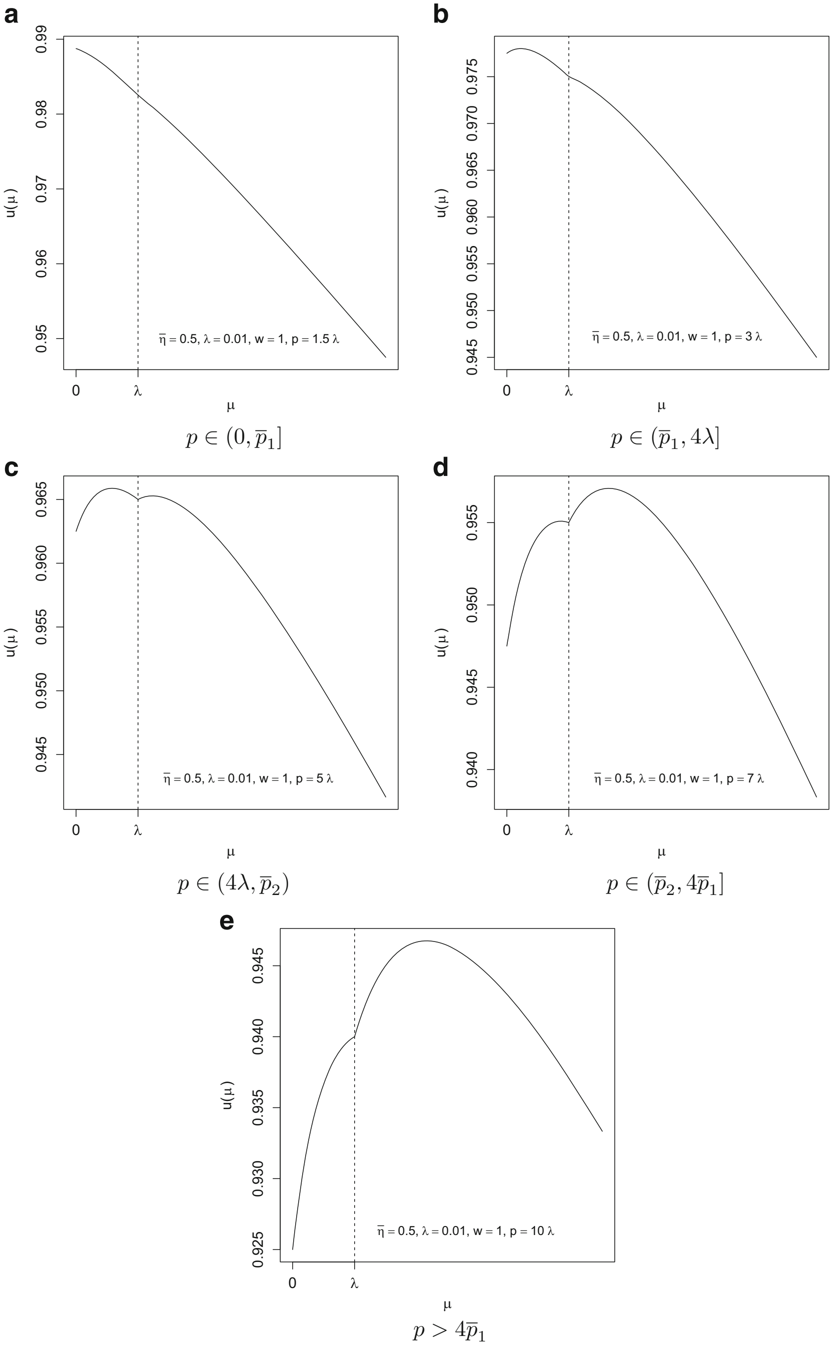


Fig. 5.2 Illustration of the forms of $u(\mu)$ when $\overline{\eta} \in (0, 3/4]$

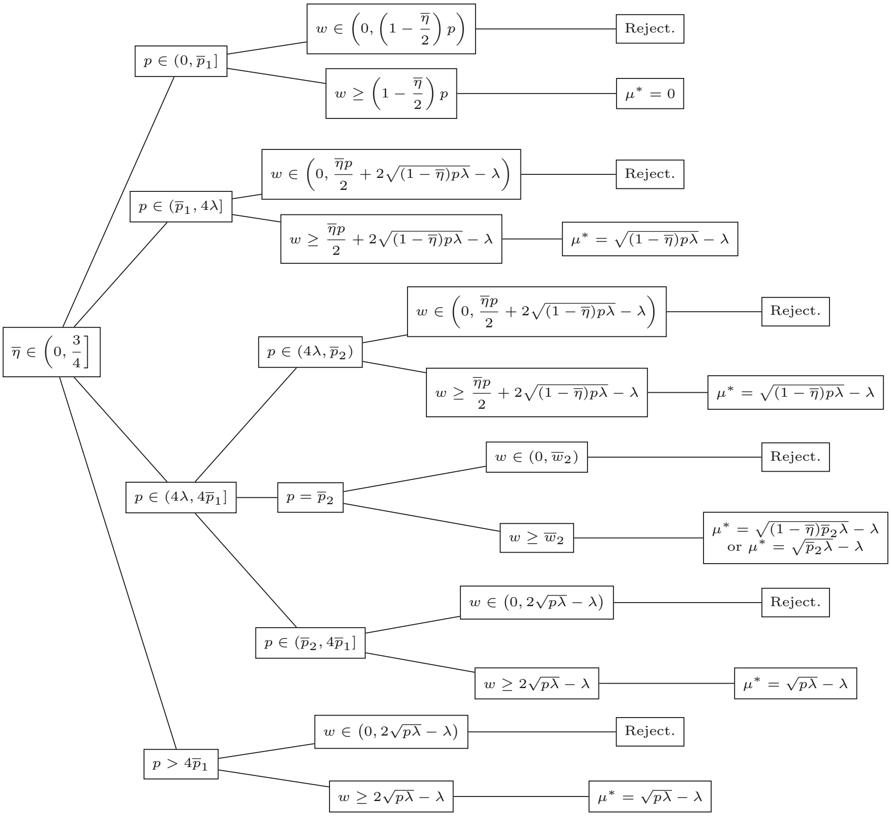


Fig. 5.3 Structure of the proof for Proposition 5.10 when $\overline{\eta} \in (0, 3/4]$

and from Eq. (5.3) $u\left(\mu^*_{(0,\lambda]}(w,p)\right) = w - \overline{\eta}p/2 - 2\sqrt{(1-\overline{\eta})p\lambda} + \lambda$. Denote the optimal service capacity for $\mu > \lambda$ by $\mu^*_\lambda(w,p)$, which is obtained from first order condition $du(\mu)/d\mu|_{\mu=\mu^*_\lambda(w,p)} = 0 \Rightarrow \mu^*_\lambda(w,p) = \sqrt{p\lambda} - \lambda$ and from Eq. (5.3) $u\left(\mu^*_\lambda(w,p)\right) = w - 2\sqrt{p\lambda} + \lambda$. The agent has a choice of two service capacities and he installs the one that generates a higher expected utility rate. Note that $u\left(\mu^*_\lambda(w,p)\right) - u\left(\mu^*_{(0,\lambda]}(w,p)\right) = \overline{\eta}p/2 - 2\left(1 - \sqrt{1-\overline{\eta}}\right)\sqrt{p\lambda}$. According to Lemma 5.9, $4\overline{p}_1 > \overline{p}_2 > 4\lambda$, therefore we examine the following subcases.

Subcase $p \in (4\lambda, \overline{p}_2)$: By Lemma 5.7 part (a), $u\left(\mu^*_{(0,\lambda]}(w,p)\right) > u\left(\mu^*_\lambda(w,p)\right)$, therefore the agent's optimal service capacity is $\mu^*(w,p) = \sqrt{(1-\overline{\eta})p\lambda} - \lambda$ and $u(\mu^*(w,p)) = w - \overline{\eta}p/2 - 2\sqrt{(1-\overline{\eta})p\lambda} + \lambda$. According to Lemmas 5.2 part (a) and 5.3, $\overline{\eta}p/2 + 2\sqrt{(1-\overline{\eta})p\lambda} - \lambda > 0$, therefore we examine the following subcases.

Subsubcase $w \in \left(0, \overline{\eta}p/2 + 2\sqrt{(1-\overline{\eta})p\lambda} - \lambda\right)$: $u(\mu^*(w,p)) < 0$, thus the agent rejects the contract.

Subsubcase $w \geq \overline{\eta}p/2 + 2\sqrt{(1-\overline{\eta})p\lambda} - \lambda$: $u(\mu^*(w,p)) \geq 0$, therefore the agent would accept the contract if offered.

Subcase $p = \overline{p}_2$: According to Lemma 5.7 part (c), $u\left(\mu^*_{(0,\lambda]}(w,p)\right) = u\big(\mu^*_\lambda(w,p)\big)$, indicating that installing either $\mu^*_{(0,\lambda]}(w,\overline{p}_2)$ or $\mu^*_\lambda(w,\overline{p}_2)$ leads to the same agent's expected utility rate. Therefore the agent is indifferent about installing $\mu^*(w,p) = \sqrt{(1-\overline{\eta})p\lambda} - \lambda$ or $\mu^*(w,p) = \sqrt{p\lambda} - \lambda$. Still, the capacity value leads to admissible solutions (see Proposition 5.13). Recall the definition of $\overline{w}_2$ from (5.6). By Lemma 5.2, $\overline{p}_2 > 4\lambda > \overline{p}_1 \Rightarrow \overline{w}_2 = \overline{\eta}\overline{p}_2/2 + 2\sqrt{(1-\overline{\eta})\overline{p}_2\lambda} - \lambda > 0$, therefore we examine the following subcases.

Subsubcase $w \in (0, \overline{w}_2)$: $u(\mu^*(w,p)) < 0$, therefore the agent rejects the contract.

Subsubcase $w \geq \overline{w}_2$: $u(\mu^*(w,p)) \geq 0$, therefore the agent would accept the contract if offered.

Subcase $p \in (\overline{p}_2, 4\overline{p}_1]$: From Lemma 5.7 part (b), $u\left(\mu^*_\lambda(w,p)\right) > u\big(\mu^*_{(0,\lambda]}(w,p)\big)$, therefore the agent's optimal service capacity is $\mu^*(w,p) = \sqrt{p\lambda} - \lambda$ and $u(\mu^*(w,p)) = w - 2\sqrt{p\lambda} + \lambda$. Since $p > \overline{p}_2 > 4\lambda \Rightarrow 2\sqrt{p\lambda} - \lambda > 3\lambda > 0$, therefore we examine the following subcases.

Subsubcase $w \in \left(0, 2\sqrt{p\lambda} - \lambda\right)$: $u(\mu^*(w,p)) < 0$, thus the agent rejects the contract.

Subsubcase $w \geq 2\sqrt{p\lambda} - \lambda$: $u(\mu^*(w,p)) \geq 0$, therefore the agent would accept the contract if offered.

Case $p > 4\overline{p}_1$: According to Table 5.1, the service capacity that maximizes $u(\mu)$ satisfies $\mu > \lambda$. From the first order condition the agent's optimal service capacity is $\mu^*(w,p) = \sqrt{p\lambda} - \lambda$ and from Eq. (5.3) $u(\mu^*(w,p)) = w - 2\sqrt{p\lambda} + \lambda$. Since $p > 4\overline{p}_1 > 4\lambda$, therefore $2\sqrt{p\lambda} - \lambda > 3\lambda > 0$ and we examine the following subcases.

Subcase $w \in \left(0, 2\sqrt{p\lambda} - \lambda\right)$: $u(\mu^*(w,p)) < 0$, thus the agent rejects the contract.

Subcase $w \geq 2\sqrt{p\lambda} - \lambda$: $u(\mu^*(w,p)) \geq 0$, therefore the agent would accept the contract if offered.

This completes the proof for Proposition 5.10 when $\overline{\eta} \in (0, 3/4]$.

Case $\overline{\eta} \in (3/4, 8/9)$: According to Lemmas 5.8 and 5.9, $4\overline{p}_1 > \overline{p}_2 > \overline{p}_1 > 4\lambda > 0$. Figure 5.4 depicts the shape of $u(\mu)$ when $\overline{\eta} \in (3/4, 8/9)$ and the value of p falls in different ranges. The structure of the proof when $\overline{\eta} \in (3/4, 8/9)$ is depicted in Fig. 5.5.

Case $p \in (0, 4\lambda]$: According to Table 5.1, $u(\mu)$ is decreasing with respect to $\mu \geq 0$. Therefore the agent's optimal service capacity is $\mu^*(w,p) = 0$ and from Eq. (5.3) $u(\mu^*(w,p)) = w - (1 - \overline{\eta}/2)p$. Note that $1 - \overline{\eta}/2 > 0$.

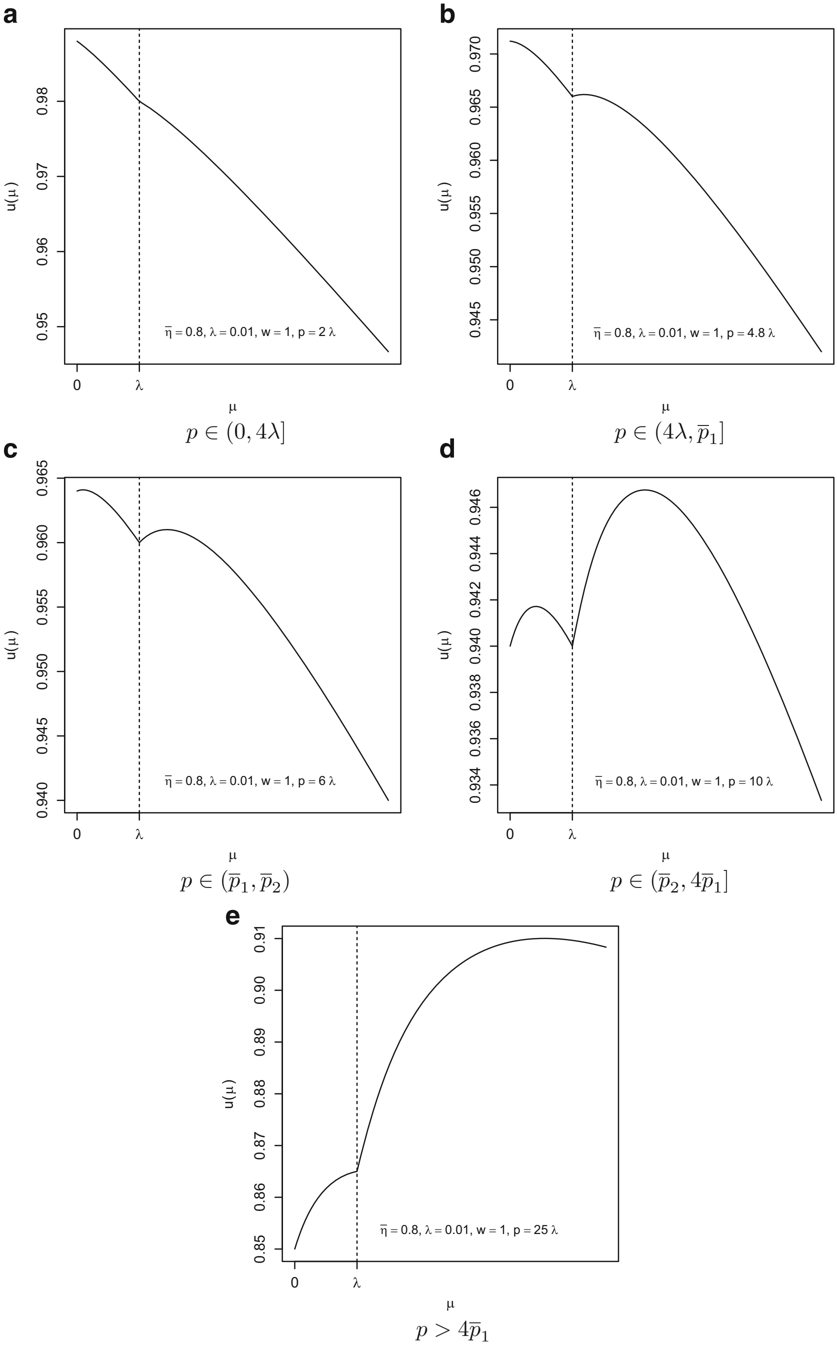


Fig. 5.4 Illustration of the forms of $u(\mu)$ when $\overline{\eta} \in (3/4, 8/9)$

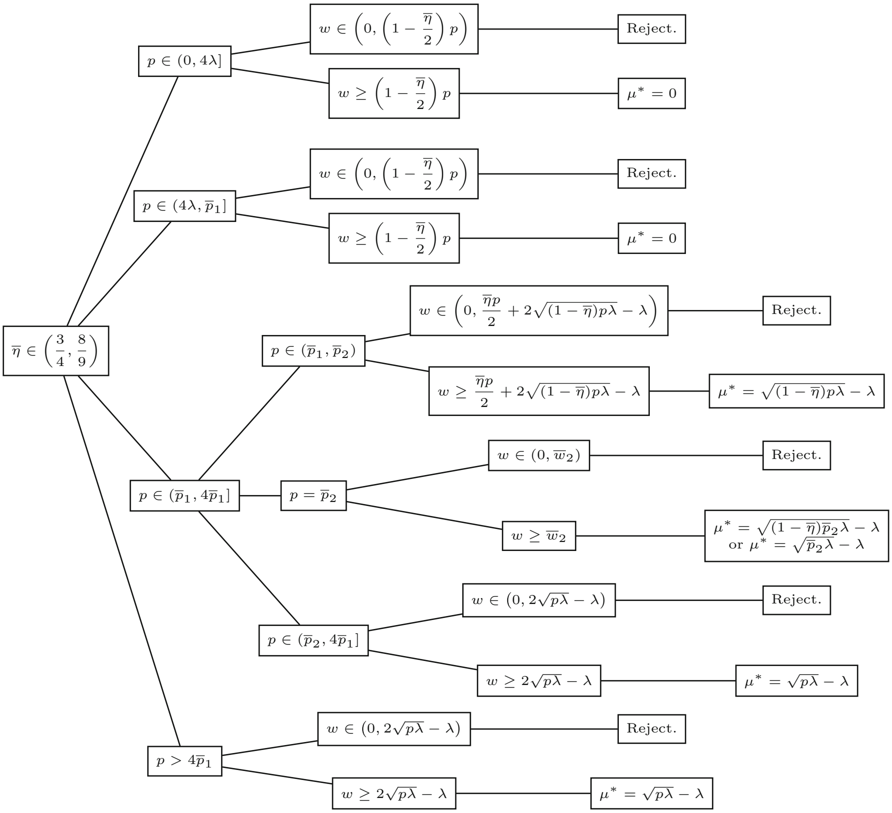


Fig. 5.5 Structure of the proof for Proposition 5.10 when $\overline{\eta} \in (3/4, 8/9)$

Subcase $w \in \left(0, (1 - \overline{\eta}/2)p\right)$: $u(\mu^*(w, p)) < 0$, therefore the agent rejects the contract.

Subcase $w \geq (1 - \overline{\eta}/2)p$: $u(\mu^*(w, p)) \geq 0$, thus the agent would accept the contract if offered.

Case $p \in \left(4\lambda, \overline{p}_1\right]$: According to Table 5.1, there is a service capacity that maximizes $u(\mu)$ for $\mu \in [0, \lambda)$ and a service capacity that maximizes $u(\mu)$ for $\mu > \lambda$. Denote the optimal service capacity in $[0, \lambda)$ by $\mu^*_{[0,\lambda)}(w, p)$. Since $u(\mu)$ is decreasing with respect to μ over $[0, \lambda)$, therefore $\mu^*_{[0,\lambda)}(w, p) = 0$ and from (5.3) $u\left(\mu^*_{[0,\lambda)}(w, p)\right) = w - (1 - \overline{\eta}/2)p$. Denote the optimal service capacity for $\mu > \lambda$ by $\mu^*_\lambda(w, p)$. From first order condition $\mu^*_\lambda(w, p) = \sqrt{p\lambda} - \lambda$ and from (5.3) $u\left(\mu^*_\lambda(w, p)\right) = w - 2\sqrt{p\lambda} + \lambda$. The agent has to choose one of the two service capacities and he installs the one with higher expected utility rate. Note that $u\left(\mu^*_\lambda(w, p)\right) - u\left(\mu^*_{[0,\lambda)}(w, p)\right) = (1 - \overline{\eta}/2)p - 2\sqrt{p\lambda} +$

λ. According to Lemma 5.5, $4\lambda > 2\lambda/\left(2+\overline{\eta}+2\sqrt{2\overline{\eta}}\right)$ and according to Lemma 5.6, $2\lambda/\left(2+\overline{\eta}-2\sqrt{2\overline{\eta}}\right) > \overline{p}_1$. Therefore according to Lemma 5.4 part (a), $u\left(\mu^*_{[0,\lambda)}(w,p)\right) > u\left(\mu^*_{\lambda}(w,p)\right)$, the agent's optimal service capacity is $\mu^*(w,p)=0$ and $u(\mu^*(w,p)) = w-(1-\overline{\eta}/2)p$. Note that $1-\overline{\eta}/2>0$, therefore we examine the following subcases.

Subcase $w \in (0,(1-\overline{\eta}/2)p)$: $u(\mu^*(w,p)) < 0$, therefore the agent rejects the contract.

Subcase $w \geq (1-\overline{\eta}/2)p$: $u(\mu^*(w,p)) \geq 0$, thus the agent would accept the contract if offered.

Case $p \in (\overline{p}_1, 4\overline{p}_1]$: According to Table 5.1, there is a service capacity that maximizes $u(\mu)$ for $\mu \in (0,\lambda]$ and a service capacity that maximizes $u(\mu)$ for $\mu > \lambda$. Denote the optimal service capacity in $(0,\lambda]$ by $\mu^*_{(0,\lambda]}(w,p)$. From first order condition the optimal service capacity is $\mu^*_{(0,\lambda]}(w,p) = \sqrt{(1-\overline{\eta})p\lambda}-\lambda$ and from (5.3) $u\left(\mu^*_{(0,\lambda]}(w,p)\right) = w-\overline{\eta}p/2-2\sqrt{(1-\overline{\eta})p\lambda}+\lambda$. Denote the optimal service capacity for $\mu > \lambda$ by $\mu^*_{\lambda}(w,p)$. From first order condition $\mu^*_{\lambda}(w,p) = \sqrt{p\lambda}-\lambda$ and from (5.3) $u\left(\mu^*_{\lambda}(w,p)\right) = w-2\sqrt{p\lambda}+\lambda$. The agent has a choice of two service capacities and he installs the one that generates a higher expected utility rate. Note that $u\left(\mu^*_{\lambda}(w,p)\right) - u\left(\mu^*_{(0,\lambda]}(w,p)\right) = \overline{\eta}p/2-2\left(1-\sqrt{1-\overline{\eta}}\right)\sqrt{p\lambda}$. According to Lemmas 5.8 and 5.9, $4\overline{p}_1 > \overline{p}_2 > \overline{p}_1$, therefore we examine the following subcases.

Subcase $p \in (\overline{p}_1, \overline{p}_2)$: By Lemma 5.7 part (a), $u\left(\mu^*_{(0,\lambda]}(w,p)\right) > u\left(\mu^*_{\lambda}(w,p)\right)$, therefore the agent's optimal service capacity is $\mu^*(w,p) = \sqrt{(1-\overline{\eta})p\lambda}-\lambda$ and $u(\mu^*(w,p)) = w-\overline{\eta}p/2-2\sqrt{(1-\overline{\eta})p\lambda}+\lambda$. According to Lemmas 5.2 and 5.3, $p > \overline{p}_1 \Rightarrow \overline{\eta}p/2+2\sqrt{(1-\overline{\eta})p\lambda}-\lambda > 0$, therefore we examine the following subcases.

Subsubcase $w \in \left(0, \overline{\eta}p/2+2\sqrt{(1-\overline{\eta})p\lambda}-\lambda\right)$: $u(\mu^*(w,p)) < 0$, thus the agent rejects the contract.

Subsubcase $w \geq \overline{\eta}p/2+2\sqrt{(1-\overline{\eta})p\lambda}-\lambda$: $u(\mu^*(w,p)) \geq 0$, therefore the agent would accept the contract if offered.

Subcase $p = \overline{p}_2$: According to Lemma 5.7 part (c), $u\left(\mu^*_{(0,\lambda]}(w,p)\right) = u\left(\mu^*_{\lambda}(w,p)\right)$, indicating that installing $\mu^*_{(0,\lambda]}(w,\overline{p}_2)$ or $\mu^*_{\lambda}(w,\overline{p}_2)$ leads to the same agent's expected utility rate. Therefore the agent is indifferent about installing $\mu^*(w,p) = \sqrt{(1-\overline{\eta})p\lambda}-\lambda$ or $\mu^*(w,p) = \sqrt{p\lambda}-\lambda$. Still, the capacity value has to lead to admissible solutions (see Proposition 5.13).

Recall the definition of $\overline{w}_2$ in (5.6). According to Lemma 5.2, $\overline{p}_2 > \overline{p}_1 \Rightarrow \overline{w}_2 = \overline{\eta}\overline{p}_2/2 + 2\sqrt{(1-\overline{\eta})\overline{p}_2\lambda} - \lambda > 0$, therefore we examine the following subcases.

Subsubcase $w \in (0, \overline{w}_2)$: $u(\mu^*(w,p)) < 0$, therefore the agent rejects the contract.

Subsubcase $w \geq \overline{w}_2$: $u(\mu^*(w,p)) \geq 0$, so the agent would accept the contract if offered.

Subcase $p \in (\overline{p}_2, 4\overline{p}_1]$: By Lemma 5.7 part (b), $u\left(\mu^*_\lambda(w,p)\right) > u\left(\mu^*_{(0,\lambda]}(w,p)\right)$, therefore the agent's optimal service capacity is $\mu^*(w,p) = \sqrt{p\lambda} - \lambda$ and $u(\mu^*(w,p)) = w - 2\sqrt{p\lambda} + \lambda$. Since $p > \overline{p}_2 > 4\lambda \Rightarrow 2\sqrt{p\lambda} - \lambda > 3\lambda > 0$, therefore we examine the following subcases.

Subsubcase $w \in \left(0, 2\sqrt{p\lambda} - \lambda\right)$: $u(\mu^*(w,p)) < 0$, thus the agent rejects the contract.

Subsubcase $w \geq 2\sqrt{p\lambda} - \lambda$: $u(\mu^*(w,p)) \geq 0$, therefore the agent would accept the contract if offered.

Case $p > 4\overline{p}_1$: According to Table 5.1, the service capacity that maximizes $u(\mu)$ satisfies $\mu > \lambda$. From the first order condition the agent's optimal service capacity is $\mu^*(w,p) = \sqrt{p\lambda} - \lambda$ and from (5.3) $u(\mu^*(w,p)) = w - 2\sqrt{p\lambda} + \lambda$. Since $p > 4\overline{p}_1 > 4\lambda$, therefore $2\sqrt{p\lambda} - \lambda > 3\lambda > 0$ and we examine the following subcases.

Subcase $w \in \left(0, 2\sqrt{p\lambda} - \lambda\right)$: $u(\mu^*(w,p)) < 0$, thus the agent rejects the contract.

Subcase $w \geq 2\sqrt{p\lambda} - \lambda$: $u(\mu^*(w,p)) \geq 0$, therefore the agent would accept the contract if offered.

This complete the proof for Proposition 5.10 when $\overline{\eta} \in (3/4, 8/9)$. □

To summarize: Given exogenous market conditions that enable a mutually beneficial contract between a principal and weakly risk-seeking agent (see Theorem 5.17 later), the agent determines his service capacity by using one of only two formulas:

$$\mu^* = \sqrt{(1-\overline{\eta})p\lambda} - \lambda > 0 \text{ or } \mu^*(w,p) = \sqrt{p\lambda} - \lambda > 0$$

The conditions when a weakly risk-seeking agent accepts the contract can be depicted by the shaded areas in Fig. 5.6, where $\overline{\eta} = 0.5$. The three shaded areas with different grey scales represent conditions (5.7), (5.8) and (5.10) under which the agent accepts the contract but responds differently. The lower bound function of the shaded area (denoted by $w_0(p)$) represents the set of offers of zero expected utility rate for the agent. The $w_0(p)$ line is defined as follows:

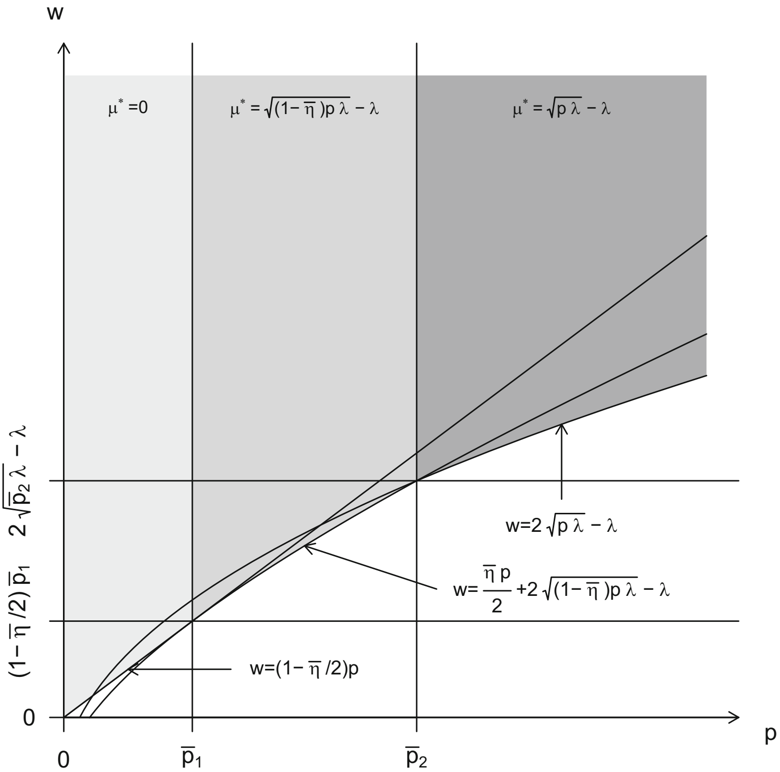


Fig. 5.6 Conditions when a weakly risk-seeking agent accepts the contract with $\overline{\eta} = 0.5$

$$w_0(p) = \begin{cases} \left(1 - \dfrac{\overline{\eta}}{2}\right) p \text{ when } p \in (0, \overline{p}_1] \\ \dfrac{\overline{\eta} p}{2} + 2\sqrt{(1-\overline{\eta})p\lambda} - \lambda \text{ when } p \in (\overline{p}_1, \overline{p}_2] \\ 2\sqrt{p\lambda} - \lambda \text{ when } p > \overline{p}_2 \end{cases}$$

Since $\lim_{p \to \overline{p}_1^-} w_0(p) = \lim_{p \to \overline{p}_1^+} w_0(p) = (1 - \overline{\eta}/2)\overline{p}_1$ and $\lim_{p \to \overline{p}_2^-} w_0(p) = \lim_{p \to \overline{p}_2^+} w_0(p) = \overline{\eta}\overline{p}_2/2 + 2\sqrt{(1-\overline{\eta})\overline{p}_2\lambda} - \lambda$, therefore $w_0(p)$ is continuous everywhere over interval $p \in \mathbb{R}_+$. Since $\lim_{p \to \overline{p}_1^-} dw_0(p)/dp = \lim_{p \to \overline{p}_1^+} dw_0(p)/dp = 1 - \overline{\eta}/2$, therefore $w_0(p)$ is differentiable at $p = \overline{p}_1$. However since $\lim_{p \to \overline{p}_2^-} dw_0(p)/dp = \overline{\eta}\left(2 - \sqrt{1-\overline{\eta}}\right)/4\left(1 - \sqrt{1-\overline{\eta}}\right) \neq \overline{\eta}/4\left(1 - \sqrt{1-\overline{\eta}}\right) = \lim_{p \to \overline{p}_2^+} dw_0(p)/dp$, therefore $w_0(p)$ is not differentiable at $p = \overline{p}_2$.

5.1.1 Sensitivity Analysis of a Weakly Risk-Seeking Agent's Optimal Strategy

A principal does not propose a contract that will be accepted by the agent but results in zero service capacity. Therefore the only viable cases when the agent accepts the contract and installs positive service capacities are: $\mu^*(w,p) = \sqrt{(1-\overline{\eta})p\lambda} - \lambda$ or $\mu^*(w,p) = \sqrt{p\lambda} - \lambda$.

First the case when a weakly risk-seeking agent installs $\mu^*(w,p) = \sqrt{(1-\overline{\eta})p\lambda} - \lambda$. According to (5.8) the compensation rate w is bounded below by $\overline{\eta}p/2 + 2\sqrt{(1-\overline{\eta})p\lambda} - \lambda = pP(1) - \overline{\eta}p\,(P(1) - 1/2) + \mu^*(w,p)$, with the term $pP(1)$ representing the expected penalty rate charged by the principal and the term $\overline{\eta}p\,(P(1) - 1/2)$ representing the expected risk rate perceived by the agent when the optimal capacity is installed. It dictates that the agent be reimbursed for the expected penalty rate and the cost of the optimal service capacity discounted by his perceived risk rate in exchange.

The optimal service capacity $\sqrt{(1-\overline{\eta})p\lambda} - \lambda$ depends on p, λ, and $\overline{\eta}$. Its derivatives are:

$$\frac{\partial\mu^*}{\partial p} = \sqrt{\frac{(1-\overline{\eta})\lambda}{4p}} > 0,\ \frac{\partial\mu^*}{\partial\lambda} = \sqrt{\frac{(1-\overline{\eta})p}{4\lambda}} - 1 \text{ and } \frac{\partial\mu^*}{\partial\overline{\eta}} = -\sqrt{\frac{p\lambda}{4(1-\overline{\eta})}} < 0$$

The above derivatives indicate that given a λ and η the agent will increase the service capacity when the penalty rate increases. Note that $\sqrt{(1-\overline{\eta})p\lambda} - \lambda$, as a function of λ, decreases when $\lambda > (1-\overline{\eta})p/4$. From conditions (5.8) and (5.9) the agent installs service capacity $\sqrt{(1-\overline{\eta})p\lambda} - \lambda$ when $p \in (\overline{p}_1, \overline{p}_2]$, and according to Lemma 5.9 we have $4\overline{p}_1 > \overline{p}_2$. Therefore we have $4\lambda/(1-\overline{\eta}) = 4\overline{p}_1 > p \Rightarrow \lambda > (1-\overline{\eta})p/4 \Rightarrow 0 > \partial\mu^*/\partial\lambda$. Thus, given the penalty rate and the risk coefficient, the agent will decrease the service capacity when the failure rate increases. Given a penalty rate and a failure rate, the agent will reduce the service capacity when he is more risk-seeking.

The agent's optimal expected utility rate when installing capacity $\mu^*(w,p) = \sqrt{(1-\overline{\eta})p\lambda} - \lambda$ is $u_A^* \equiv u_A(\mu^*(w,p); w,p) = w - \overline{\eta}p/2 - 2\sqrt{(1-\overline{\eta})p\lambda} + \lambda$, and it depends on w, p, $\overline{\eta}$ and λ. Note that $\partial u_A^*/\partial w = -1 < 0$, $\partial u_A^*/\partial p = -\overline{\eta}/2 - \sqrt{(1-\overline{\eta})\lambda/p} < 0$, indicating that the agent's optimal expected utility rate decreases with the compensation rate and the penalty rate. Note that $\partial u_A^*/\partial\overline{\eta} = -\sqrt{p}\left(\sqrt{p} - \sqrt{4\overline{p}_1}\right)/2$. From Proposition 5.10 $p < \overline{p}_2 < 4\overline{p}_1 \Rightarrow \sqrt{p} < \sqrt{4\overline{p}_1}$, therefore the agent's optimal expected utility rate increases with his risk intensity. Note that $\partial u_A^*/\partial\lambda = -\left(\sqrt{p} - \sqrt{\overline{p}_1}\right)/\sqrt{\overline{p}_1}$, and from Proposition 4.23 $p > \overline{p}_1 \Rightarrow \sqrt{p} - \sqrt{\overline{p}_1} > 0$, therefore the agent's optimal expected utility rate decreases with the failure rate.

Then the case when a weakly risk-seeking agent installs $\mu^*(w,p) = \sqrt{p\lambda} - \lambda$. In this case the agent's optimal strategy is identical to the optimal strategy when he is risk-neutral. According to (5.10) the w is bounded below by $2\sqrt{p\lambda} - \lambda = pP(1) +$

$\mu^*(w,p)$, with the term $pP(1)$ representing the expected penalty rate charged by the principal. It indicates that the agent will have to be reimbursed for the expected penalty rate and the cost of the optimal service capacity.

The optimal service capacity $\sqrt{p\lambda} - \lambda$ depends on the penalty rate p and the failure rate λ. Its derivatives are $\partial\mu^*/\partial p = \sqrt{\lambda/4p} > 0$ and $\partial\mu^*/\partial\lambda = \sqrt{p/4\lambda} - 1$. These derivatives imply that given λ, the agent will increase the service capacity when the penalty rate increases. Note that $\sqrt{p\lambda} - \lambda$, as a function of λ, increases when $p/4 > \lambda$. From conditions (5.9) and (5.10) the agent installs service capacity $\sqrt{p\lambda} - \lambda$ when $p \geq \overline{p}_2$, and according to Lemma 5.9 we have $\overline{p}_2 > 4\lambda$. Therefore we have $p > 4\lambda \Rightarrow p/4 > \lambda \Rightarrow \partial\mu^*/\partial\lambda > 0$. Thus, given p, an agent will increase μ when λ increases.

The agent's optimal expected utility rate when installing capacity $\mu^*(w,p) = \sqrt{p\lambda} - \lambda$ is $u_A^* \equiv u_A(\mu^*(w,p); w,p) = w - 2\sqrt{p\lambda} + \lambda$, and it depends on w, p and λ only. Note that $\partial u_A^*/\partial w = -1 < 0$, $\partial u_A^*/\partial p = -\sqrt{\lambda/p} < 0$, indicating that the agent's optimal expected utility rate decreases with the compensation rate and the penalty rate. Note that $\partial u_A^*/\partial\lambda = -\sqrt{p/\lambda} + 1$, and from Proposition 5.10 $p \geq \overline{p}_2 > 4\lambda \Rightarrow -\sqrt{p/\lambda} + 1 < 0$, therefore the agent's optimal expected utility rate also decreases with the failure rate.

Summary: Recall that given the set of contract offers $\{(w,p) : p \in (0,\lambda], w \geq p\}$ a risk-neutral agent would accept the contract, install $\mu^*(w,p) = 0$ and receive expected utility rate $u(\mu^*(w,p); w,p) = w - p$. Given the set of offers $\left\{(w,p) : p > \lambda, w \geq 2\sqrt{p\lambda} - \lambda\right\}$ he would accept the contract, install $\mu^*(w,p) = \sqrt{p\lambda} - \lambda$ and receive expected utility rate $u(\mu^*(w,p); w,p) = w - 2\sqrt{p\lambda} + \lambda$. By comparing the optimal capacities of a weakly risk-seeking agent to that of a risk-neutral agent, three conclusions are drawn.

1. The principal has to set a higher penalty rate p in order to induce a weakly risk-seeking agent to install a positive service capacity versus a risk-neutral agent ($p > \lambda$ for risk-neutral agent, $p > \lambda/(1-\overline{\eta})$ for weakly risk-seeking agent).
2. When p is relatively low, μ plays a more prominent role in the utility of a weakly risk-seeking agent who therefore installs a μ lower than that when he is risk-neutral ($\sqrt{p\lambda} - \lambda > \sqrt{(1-\overline{\eta})p\lambda} - \lambda$). As p increases, the weakly risk-seeking agent installs μ that is identical to the one for risk-neutral agent ($\sqrt{p\lambda} - \lambda$).
3. Weakly risk-seeking agent is not worse off.

This conclusion is restated in Proposition 5.11.

Proposition 5.11. *Given w and p, an agent who accepts the contract and installs a positive service capacity has a non-decreasing expected utility rate with $\overline{\eta}$ for $\overline{\eta} \in [0, 8/9)$.*

Proof. Recall that when the compensation rate w and the penalty rate p satisfy conditions (5.8) and (5.9), the agent installs service capacity $\mu^*(w,p) = \sqrt{(1-\overline{\eta})p\lambda} - \lambda > 0$, and the agent's expected utility rate is $u(\mu^*(w,p)) = w - \overline{\eta}p/2 - 2\sqrt{(1-\overline{\eta})p\lambda} + \lambda$. Note that $\partial u/\partial\overline{\eta} = -p/2 + \sqrt{p\lambda/(1-\overline{\eta})} =$

$-\sqrt{p}\left(\sqrt{p}-\sqrt{4\overline{p}_1}\right)/2$. According to Lemma 5.9, $4\overline{p}_1 > \overline{p}_2 \geq p$, therefore $\partial u/\partial\overline{\eta} > 0$. When the compensation rate w and the penalty rate p satisfy conditions (5.9) and (5.10), the agent installs service capacity $\mu^*(w,p) = \sqrt{p\lambda} - \lambda > 0$, and the agent's expected utility rate is $u(\mu^*(w,p)) = w - 2\sqrt{p\lambda} + \lambda$, therefore $\partial u/\partial\overline{\eta} = 0$. □

Corollary 5.12. *Given w and p, an agent who accepts the contract and subsequently installs a positive service capacity will not be worse off when he is weakly risk-seeking ($\overline{\eta} \in (0, 8/9)$) than risk-neutral ($\overline{\eta} = 0$).*

We return to the case of $\overline{\eta} \geq 8/9$ in Sects. 5.2.1 and 5.3.

5.1.2 Principal's Optimal Strategy

We now proceed to derive the principal's optimal strategy. Anticipating the agent's optimal selection of $\mu^*(w,p)$ the principal chooses w and p that maximize her expected profit rate by solving the optimization problem

$$\max_{w>0,p>0} \Pi_P(w,p;\mu^*(w,p)) = \max_{w>0,p>0}\left\{\frac{r\mu^*(w,p)}{\lambda+\mu^*(w,p)} - w + \frac{p\lambda}{\lambda+\mu^*(w,p)}\right\} \tag{5.11}$$

Denote $(w^*, p^*) = \text{argmax}_{w>0,p>0}\, \Pi_P(w,p;\mu^*(w,p))$.

Before deriving the principal's optimal strategy, we examine the case when the principal offers $p = \overline{p}_2$ and $w \geq \overline{w}_2$, under which the agent is indifferent about installing two different service capacities. In such a case, the solution $((w,p),\mu)$ has to be an admissible solution (see Definition 5.1). We state this case formally in Proposition 5.13.

Proposition 5.13. *Suppose a weakly risk-seeking agent. Assume that the principal's possible offers are constrained to set $\{(w,p) : p = \overline{p}_2, w \geq \overline{w}_2\}$.*

(a) *If $r \in (0,\overline{p}_2)$, then the agent installs $\mu^* = \sqrt{(1-\overline{\eta})\overline{p}_2\lambda} - \lambda$ if offered a contract.*
(b) *If $r = \overline{p}_2$, then both $\mu^* = \sqrt{(1-\overline{\eta})\overline{p}_2\lambda} - \lambda$ and $\mu^* = \sqrt{\overline{p}_2\lambda} - \lambda$ lead to admissible solutions and the agent installs either $\sqrt{(1-\overline{\eta})\overline{p}_2\lambda} - \lambda$ or $\sqrt{\overline{p}_2\lambda} - \lambda$ if offered a contract.*
(c) *If $r > \overline{p}_2$, then the agent installs $\mu^* = \sqrt{\overline{p}_2\lambda} - \lambda$ if offered a contract.*

Proof. Note that for $w \geq \overline{w}_2$ we have $\partial\Pi_P(w,\overline{p}_2;\mu)/\partial\mu = (r-\overline{p}_2)\lambda/(\lambda+\mu)^2$. Define $\mu_L \equiv \sqrt{(1-\overline{\eta})\overline{p}_2\lambda} - \lambda$ and $\mu_H \equiv \sqrt{\overline{p}_2\lambda} - \lambda$. Note that $\mu_H > \mu_L$. If $r \in (0,\overline{p}_2)$, then $\partial\Pi_P/\partial\mu < 0$, therefore $((w,\overline{p}_2),\mu_L) \succeq ((w,\overline{p}_2),\mu_H)$. If the principal offers a contract (the conditions are discussed in detail in Theorem 5.17 that follows), then by Definition 5.1 only μ_L leads to admissible solutions and we obtain (a). If $r > \overline{p}_2$, then $\partial\Pi_P/\partial\mu > 0$, therefore $((w,\overline{p}_2),\mu_H) \succeq ((w,\overline{p}_2),\mu_L)$.

If the principal offers a contract (the conditions are discussed in Theorem 5.17 that follows), then by Definition 5.1 only μ_H leads to admissible solutions and we obtain (c). If $r = \overline{p}_2$, then $\partial \Pi_P / \partial \mu = 0$, indicating that the principal receives the same expected profit rate when the agent installs capacity μ_L or μ_H. If the principal offers a contract (the conditions are discussed in Theorem 5.17 that follows), then both μ_L and μ_H lead to admissible solutions. Therefore we obtain (b). □

Notation:

$$\overline{r}_1 \equiv \overline{\eta}\overline{p}_2 + (1-\overline{\eta})\sqrt{\overline{p}_1 \overline{p}_2} - \frac{\overline{\eta}\overline{p}_2}{2}\left(\frac{\sqrt{\overline{p}_2}}{\sqrt{\overline{p}_2} - \sqrt{\overline{p}_1}}\right), \overline{r}_2 \equiv (1-\overline{\eta})\overline{p}_2 + \overline{\eta}\overline{p}_2\left(\frac{\sqrt{\overline{p}_2} - \sqrt{\overline{p}_1}}{\sqrt{\overline{p}_1}}\right) \tag{5.12}$$

Note that $\overline{r}_1$ and $\overline{r}_2$ are functions of λ and $\overline{\eta}$. However we suppress the parameters $(\lambda, \overline{\eta})$.

We define $\overline{p}_{cu}$ as follows[1]:

$$\overline{p}_{cu} \equiv \frac{1}{9a^2}\left(b + C + \overline{C}\right)^2 \tag{5.13}$$

where $a \equiv \overline{\eta}$, $b \equiv (1-2\overline{\eta})\sqrt{\overline{p}_1}$, and $d \equiv -r\sqrt{\overline{p}_1}$ and

$$C \equiv \sqrt[3]{\frac{\Delta_1 + \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}}, \overline{C} \equiv \sqrt[3]{\frac{\Delta_1 - \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}}, \text{ where } \Delta_0 \equiv b^2, \Delta_1 \equiv 2b^3 + 27a^2 d$$

Replacing Δ_0 and Δ_1 by the expressions of a, b and d we have

$$C = \sqrt[3]{\frac{2(1-2\overline{\eta})^3\sqrt{\overline{p}_1^3} - 27\overline{\eta}^2 r\sqrt{\overline{p}_1} + \sqrt{-108\overline{\eta}^2 r(1-2\overline{\eta})^3\overline{p}_1^2 + 729\overline{\eta}^4 r^2 \overline{p}_1}}{2}} \text{ and}$$

$$\overline{C} = \sqrt[3]{\frac{2(1-2\overline{\eta})^3\sqrt{\overline{p}_1^3} - 27\overline{\eta}^2 r\sqrt{\overline{p}_1} - \sqrt{-108\overline{\eta}^2 r(1-2\overline{\eta})^3\overline{p}_1^2 + 729\overline{\eta}^4 r^2 \overline{p}_1}}{2}}$$

Next we state a number of technical lemmas (see proofs in the Appendix).

Lemma 5.14. *Let* $8/9 > \overline{\eta} > 0$ *and* $\lambda > 0$, *then*

[1]The subscript "*cu*" stands for "cubic" because (5.13) is the square of the solution to Eq. (A.2), which is a cubic equation that is introduced later in the proof for Lemma 5.15.

(a) $\overline{p}_2 > (1-\overline{\eta})\overline{p}_2 + \overline{\eta}\overline{p}_2\left(\frac{\sqrt{\overline{p}_2}-\sqrt{\overline{p}_1}}{\sqrt{\overline{p}_1}}\right) > \overline{\eta}\overline{p}_2 + (1-\overline{\eta})\sqrt{\overline{p}_1\overline{p}_2} - \frac{\overline{\eta}\overline{p}_2}{2}\left(\frac{\sqrt{\overline{p}_2}}{\sqrt{\overline{p}_2}-\sqrt{\overline{p}_1}}\right)$.

(b) $(1-\overline{\eta})\overline{p}_2 + \overline{\eta}\overline{p}_2\left(\frac{\sqrt{\overline{p}_2}-\sqrt{\overline{p}_1}}{\sqrt{\overline{p}_1}}\right) > \lambda$.

Lemma 5.15. *Consider* $\max_{x\in[\sqrt{\overline{p}_1},\sqrt{\overline{p}_2}]} f(x)$ *where* $f(x) = r + \lambda - \overline{\eta}x^2/2 - \sqrt{\overline{p}_1}\,((1-2\overline{\eta})x + r/x)$ *and denote* $x^* = \operatorname{argmax}_{x\in[\sqrt{\overline{p}_1},\sqrt{\overline{p}_2}]} f(x)$. *The solutions to this optimization problem are:*

(a) $x^* = \sqrt{\overline{p}_1}$ *if* $r \in (0,\lambda]$.
(b) $x^* = \sqrt{\overline{p}_{cu}} \in \left(\sqrt{\overline{p}_1}, \sqrt{\overline{p}_2}\right)$ *if* $r \in (\lambda, \overline{r}_2)$.
(c) $x^* = \sqrt{\overline{p}_2}$ *if* $r \geq \overline{r}_2$.

Lemma 5.16. *Consider* $\max_{x\geq\sqrt{\overline{p}_2}} f(x)$ *where* $f(x) = r + \lambda - \sqrt{\lambda}\,(x + r/x)$ *and denote* $x^* = \operatorname{argmax}_{x\geq\sqrt{\overline{p}_2}} f(x)$. *Solutions to this optimization problem are*

(a) $x^* = \sqrt{\overline{p}_2}$ *if* $r \in (0, \overline{p}_2]$.
(b) $x^* = \sqrt{r}$ *if* $r > \overline{p}_2$.

Lemma 5.14 implies $\overline{p}_2 > \overline{r}_2 > \overline{r}_1$ and $\overline{r}_2 > \lambda$.

Recall that Proposition 5.10 describes the agent's optimal response to each pair $(w,p) \in \mathbb{R}^2_+$. Since the principal will not propose a contract that ex ante is going to be rejected by a weakly risk-seeking (WRS) agent, therefore Theorem 5.17 only considers pairs $(w,p) \in \mathbb{R}^2_+$ that result in agent's non-negative expected utility rate. Define

$$\begin{aligned}
\mathfrak{D}_{(5.7)} &\equiv \{(w,p) \text{ that satisfies (5.7) when } \overline{\eta} \in (0, 8/9)\}\\
\mathfrak{D}_{(5.8)} &\equiv \{(w,p) \text{ that satisfies (5.8) when } \overline{\eta} \in (0, 8/9)\}\\
\mathfrak{D}_{(5.9)} &\equiv \{(w,p) \text{ that satisfies (5.9) when } \overline{\eta} \in (0, 8/9)\}\\
\mathfrak{D}_{(5.10)} &\equiv \{(w,p) \text{ that satisfies (5.10) when } \overline{\eta} \in (0, 8/9)\}\\
\mathfrak{D}_{\text{WRS}} &\equiv \mathfrak{D}_{(5.7)} \cup \mathfrak{D}_{(5.8)} \cup \mathfrak{D}_{(5.9)} \cup \mathfrak{D}_{(5.10)}
\end{aligned} \tag{5.14}$$

Theorem 5.17. *Given a weakly risk-seeking agent and* $(w,p) \in \mathfrak{D}_{\text{WRS}}$.

(a) *If* $r \in (0,\lambda]$, *then the principal does not propose a contract.*
(b) *If* $r \in (\lambda, \overline{r}_2)$, *then the principal's offer and the capacity installed by the agent are:*

$$(w^*,p^*) = \left(\frac{\overline{\eta}\overline{p}_{cu}}{2} + 2\sqrt{(1-\overline{\eta})\overline{p}_{cu}\lambda} - \lambda, \overline{p}_{cu}\right) \text{ and } \mu^*(w^*,p^*) = \sqrt{(1-\overline{\eta})\overline{p}_{cu}\lambda} - \lambda \tag{5.15}$$

and the principal's expected profit rate is:

$$\Pi_P(w^*,p^*;\mu^*(w^*,p^*)) = r+\lambda-\frac{\overline{\eta}\overline{p}_{cu}}{2}-\sqrt{\overline{p}_1}\left((1-2\overline{\eta})\sqrt{\overline{p}_{cu}}+\frac{r}{\sqrt{\overline{p}_{cu}}}\right) \tag{5.16}$$

(c) *If* $r \in [\overline{r}_2, \overline{p}_2]$*, then the principal's offer and the capacity installed by the agent are:*

$$\left(w^*,p^*\right) = (\overline{w}_2,\overline{p}_2) \text{ and } \mu^*(w^*,p^*) = \sqrt{(1-\overline{\eta})\overline{p}_2\lambda}-\lambda \tag{5.17}$$

and the principal's expected profit rate is:

$$\Pi_P(w^*,p^*;\mu^*(w^*,p^*)) = r+\lambda-\frac{\overline{\eta}\overline{p}_2}{2}-\sqrt{\overline{p}_1}\left((1-2\overline{\eta})\sqrt{\overline{p}_2}+\frac{r}{\sqrt{\overline{p}_2}}\right) \tag{5.18}$$

(d) *If* $r > \overline{p}_2$*, then the principal's offer and the capacity installed by the agent are*

$$\left(w^*,p^*\right) = \left(2\sqrt{r\lambda}-\lambda, r\right) \text{ and } \mu^*(w^*,p^*) = \sqrt{r\lambda}-\lambda \tag{5.19}$$

and the principal's expected profit rate is:

$$\Pi_P(w^*,p^*;\mu^*(w^*,p^*)) = r-2\sqrt{r\lambda}+\lambda \tag{5.20}$$

Proof. The structure of the proof for Theorem 5.17 is depicted in Fig. 5.7.

Case $(w,p) \in \mathfrak{D}_{(5.7)}$: According to Proposition 5.10 part (a), in case the principal makes an offer, the agent accepts the contract but does not install any service capacity. Since $\partial\Pi_P/\partial w = -1 < 0$, thus we have $w^* = (1-\overline{\eta}/2)p$ and from (3.3) $\Pi_P(w^*,p;\mu^*(w^*,p)) = -w^*+p = \overline{\eta}p/2 > 0$. However in such case $p > w^* = (1-\overline{\eta}/2)p$, which violates condition (c) in Definition 5.1, therefore $((w^* = (1-\overline{\eta}/2)p,p), \mu^* = 0)$ is not an admissible solution and the principal does not propose a contract.

Case $(w,p) \in \mathfrak{D}_{(5.8)} \cup \mathfrak{D}_{(5.9)}$: According to Proposition 5.10 part (b), if $(w,p) \in \mathfrak{D}_{(5.8)}$, then in case the principal makes an offer, the agent accepts the contract and installs $\sqrt{(1-\overline{\eta})p\lambda}-\lambda$. Since $\partial\Pi_P/\partial w = -1 < 0$, therefore $w^* = \overline{\eta}p/2 + 2\sqrt{(1-\overline{\eta})p\lambda}-\lambda$. According to Propositions 5.10 part (c) and 5.13, if $(w,p) \in \mathfrak{D}_{(5.9)}$ (which implies $p = \overline{p}_2$), then in case the principal makes an offer, the agent accepts the contract and installs $\sqrt{(1-\overline{\eta})\overline{p}_2\lambda}-\lambda$ if $r \in (0,\overline{p}_2)$, installs either $\sqrt{(1-\overline{\eta})\overline{p}_2\lambda}-\lambda$ or $\sqrt{\overline{p}_2\lambda}-\lambda$ if $r = \overline{p}_2$, or installs $\sqrt{\overline{p}_2\lambda}-\lambda$ if $r > \overline{p}_2$. Since $\partial\Pi_P/\partial w = -1 < 0$, therefore $w^* = \overline{w}_2$. For convenience denote the principal's expected profit rate when $(w,p) = (\overline{w}_2,\overline{p}_2)$ and $\mu^*(w,p) = \sqrt{(1-\overline{\eta})\overline{p}_2\lambda}-\lambda$ by

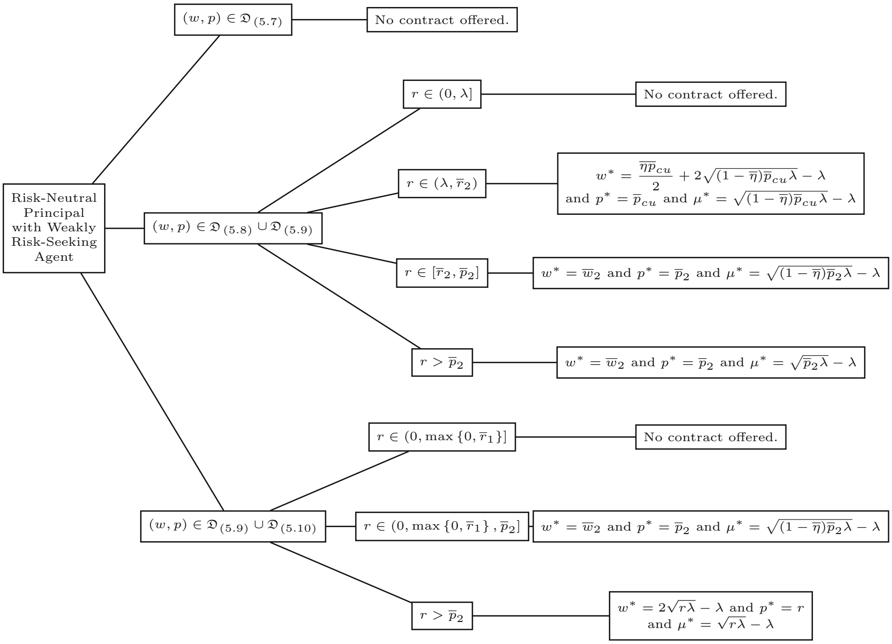


Fig. 5.7 Structure of the proof for Theorem 5.17

$\Pi_P^L(\overline{p}_2)$, and denote the principal's expected profit rate when $(w,p) = (\overline{w}_2, \overline{p}_2)$ and $\mu^*(w,p) = \sqrt{\overline{p}_2\lambda} - \lambda$ by $\Pi_P^H(\overline{p}_2)$. By plugging the value of w, p and μ into (3.3):

$$\Pi_P^L(\overline{p}_2) = r + \lambda - \frac{\overline{\eta}\overline{p}_2}{2} - \sqrt{\overline{p}_1}\left((1-2\overline{\eta})\sqrt{\overline{p}_2} + \frac{r}{\sqrt{\overline{p}_2}}\right) = \left(\frac{\sqrt{\overline{p}_2} - \sqrt{\overline{p}_1}}{\sqrt{\overline{p}_2}}\right)(r - \overline{r}_1) \tag{5.21}$$

$$\Pi_P^H(\overline{p}_2) = r + \lambda - \sqrt{\lambda}\left(\sqrt{\overline{p}_2} + \frac{r}{\sqrt{\overline{p}_2}}\right) \tag{5.22}$$

In such case the principal's optimization problem is $\max_{p\in[\overline{p}_1,\overline{p}_2]} \Pi_P(w^*, p; \mu^*(w^*,p))$ where:

$$\Pi_P(w^*, p; \mu^*(w^*,p)) = \begin{cases} r + \lambda - \frac{\overline{\eta}p}{2} - \sqrt{\overline{p}_1}\left((1-2\overline{\eta})\sqrt{p} + \frac{r}{\sqrt{p}}\right), \text{for } p \in [\overline{p}_1, \overline{p}_2) \\ \max\left\{\Pi_P^L(\overline{p}_2), \Pi_P^H(\overline{p}_2)\right\}, \text{for } p = \overline{p}_2 \end{cases}$$

Define $x \equiv \sqrt{p}$, the expression $r + \lambda - \overline{\eta}p/2 - \sqrt{\overline{p}_1}\left((1-2\overline{\eta})\sqrt{p} + r/\sqrt{p}\right)$ can be restated as $f(x) = r + \lambda - \overline{\eta}x^2/2 - \sqrt{\overline{p}_1}\left((1-2\overline{\eta})x + r/x\right)$. Maximizing $f(x)$

with respect to x over $\left[\sqrt{\overline{p}_1}, \sqrt{\overline{p}_2}\right]$ is equivalent to maximizing $r + \lambda - \overline{\eta}p/2 - \sqrt{\overline{p}_1}\left((1-2\overline{\eta})\sqrt{p} + r/\sqrt{p}\right)$ with respect to p over $[\overline{p}_1, \overline{p}_2]$ in the sense that

$$\underset{p\in[\overline{p}_1,\overline{p}_2]}{\text{argmax}}\left\{r + \lambda - \frac{\overline{\eta}p}{2} - \sqrt{\overline{p}_1}\left((1-2\overline{\eta})\sqrt{p} + \frac{r}{\sqrt{p}}\right)\right\} = \left(\underset{x\in\left[\sqrt{\overline{p}_1},\sqrt{\overline{p}_2}\right]}{\text{argmax}} f(x)\right)^2$$

From Lemma 5.14, $\overline{p}_2 > \overline{r}_2 > \lambda$ and we examine the following subcases:

Subcase $r \in (0, \lambda]$: According to Lemma 5.15 part (a), $p^* = \overline{p}_1$, which is covered in $(w, p) \in \mathfrak{D}_{(5.7)}$ and the principal does not propose a contract.

Subcase $r \in (\lambda, \overline{r}_2)$: According to Lemma 5.15 part (b), $p^* = \overline{p}_{cu}$ and the principal's expected profit rate is $\Pi_P(w^*, p^*; \mu^*(w^*, p^*)) > \Pi_P((1 - \overline{\eta}/2)\overline{p}_1, \overline{p}_1; 0) = \overline{\eta}\overline{p}_1/2 > 0$. Therefore the principal proposes a contract with $w^* = \overline{\eta}\overline{p}_{cu}/2 + 2\sqrt{(1-\overline{\eta})\overline{p}_{cu}\lambda} - \lambda$ and $p^* = \overline{p}_{cu}$ that induces the agent to install $\mu^*(w^*, p^*) = \sqrt{(1-\overline{\eta})\overline{p}_{cu}\lambda} - \lambda$.

Subcase $r \in [\overline{r}_2, \overline{p}_2]$: According to Lemma 5.15 part (c), $p^* = \overline{p}_2$ and according to Proposition 5.13 part (a) and (b) the principal's expected profit rate is $\Pi_P(w^*, p^*; \mu^*(w^*, p^*)) = \Pi_P^L(\overline{p}_2) > \Pi_P((1 - \overline{\eta}/2)\overline{p}_1, \overline{p}_1; 0) = \overline{\eta}\overline{p}_1/2 > 0$. Therefore the principal proposes a contract with $w^* = \overline{w}_2$ and $p^* = \overline{p}_2$ that induces the agent to install $\mu^*(w^*, p^*) = \sqrt{(1-\overline{\eta})\overline{p}_2\lambda} - \lambda$.

Subcase $r > \overline{p}_2$: According to Lemma 5.15 part (c), $p^* = \overline{p}_2$ and according to Proposition 5.13 part (c) her expected profit rate is $\Pi_P(w^*, p^*; \mu^*(w^*, p^*)) = \Pi_P^H(\overline{p}_2) > \Pi_P^L(\overline{p}_2) > \Pi_P((1 - \overline{\eta}/2)\overline{p}_1, \overline{p}_1; 0) = \overline{\eta}\overline{p}_1/2 > 0$. Therefore the principal proposes a contract with $w^* = \overline{w}_2$ and $p^* = \overline{p}_2$ that induces the agent to install $\mu^*(w^*, p^*) = \sqrt{\overline{p}_2\lambda} - \lambda$.

Case $(w, p) \in \mathfrak{D}_{(5.9)} \cup \mathfrak{D}_{(5.10)}$: According to Proposition 5.10 part (d), if $(w, p) \in \mathfrak{D}_{(5.10)}$, then in case the principal makes an offer, the agent accepts the contract and installs $\sqrt{p\lambda} - \lambda$. Since $\partial\Pi_P/\partial w = -1 < 0$, therefore $w^* = 2\sqrt{p\lambda} - \lambda$. According to Propositions 5.10 part (c) and 5.13, if $(w, p) \in \mathfrak{D}_{(5.9)}$ (which implies $p = \overline{p}_2$), then in case the principal makes an offer, the agent accepts the contract and installs $\sqrt{(1-\overline{\eta})\overline{p}_2\lambda} - \lambda$ if $r \in (0, \overline{p}_2)$, installs either $\sqrt{(1-\overline{\eta})\overline{p}_2\lambda} - \lambda$ or $\sqrt{\overline{p}_2\lambda} - \lambda$ if $r = \overline{p}_2$, or installs $\sqrt{\overline{p}_2\lambda} - \lambda$ if $r > \overline{p}_2$. Since $\partial\Pi_P/\partial w = -1 < 0$, therefore $w^* = \overline{w}_2$. Recall the definition of $\Pi_P^L(\overline{p}_2)$ and $\Pi_P^H(\overline{p}_2)$ (see Eqs. (5.21) and (5.22)). Thus the principal's optimization problem is $\max_{p\geq\overline{p}_2} \Pi_P(w^*, p; \mu^*(w^*, p))$ where:

$$\Pi_P(w^*, p; \mu^*(w^*, p)) = \begin{cases} \max\left\{\Pi_P^L(\overline{p}_2), \Pi_P^H(\overline{p}_2)\right\}, \text{for } p = \overline{p}_2 \\ r + \lambda - \sqrt{\lambda}\left(\sqrt{p} + \dfrac{r}{\sqrt{p}}\right), \text{for } p > \overline{p}_2 \end{cases}$$

Define $x \equiv \sqrt{p}$, the expression $r + \lambda - \sqrt{\lambda}\left(\sqrt{p} + r/\sqrt{p}\right)$ can be restated as $f(x) = r + \lambda - \sqrt{\lambda}\,(x + r/x)$. Maximizing $f(x)$ with respect to $x \geq \sqrt{\overline{p}_2}$ is equivalent to maximizing $r + \lambda - \sqrt{\lambda}\left(\sqrt{p} + r/\sqrt{p}\right)$ with respect to $p \geq \overline{p}_2$ in the sense that

$$\underset{p \geq \overline{p}_2}{\operatorname{argmax}} \left\{ r + \lambda - \sqrt{\lambda}\left(\sqrt{p} + \frac{r}{\sqrt{p}}\right) \right\} = \left(\underset{x \geq \sqrt{\overline{p}_2}}{\operatorname{argmax}} f(x) \right)^2$$

According to Lemma 5.14, $\overline{p}_2 > \overline{r}_1$. Also note that $\lim_{\overline{\eta} \to 0^+} \overline{r}_1 = 2\lambda$ and according to Lemma 5.8 $\lim_{\overline{\eta} \to 8/9^-} \overline{r}_1 = -\infty$. Therefore we examine the following subcases:

Subcase $r \in (0, \max\{0, \overline{r}_1\}]$: According to Lemma 5.16 part (a), $p^* = \overline{p}_2$. According to Proposition 5.13 part (a), $\Pi_P(w^*, p^*; \mu^*(w^*, p^*)) = \Pi_P^L(\overline{p}_2) \leq 0$, therefore the principal does not propose a contract.

Subcase $r \in (\max\{0, \overline{r}_1\}, \overline{p}_2]$: According to Lemma 5.16 part (a), $p^* = \overline{p}_2$. According to Proposition 5.13 part (a) and (b), $\Pi_P(w^*, p^*; \mu^*(w^*, p^*)) = \Pi_P^L(\overline{p}_2) > 0$, therefore the principal proposes a contract with $w^* = \overline{w}_2$ and $p^* = \overline{p}_2$ that induces the agent to install $\mu^*(w^*, p^*) = \sqrt{(1-\overline{\eta})\overline{p}_2\lambda} - \lambda$.

Subcase $r > \overline{p}_2$: According to Proposition 5.16 part (b), $p^* = r$ and the principal's expected profit rate is $\Pi_P(w^*, p^*; \mu^*(w^*, p^*)) = r - 2\sqrt{r\lambda} + \lambda > 0$. Thus the principal proposes a contract with $w^* = 2\sqrt{r\lambda} - \lambda$ and $p^* = r$ that induces the agent to install $\mu^*(w^*, p^*) = \sqrt{r\lambda} - \lambda$.

To summarize, if $r \in (0, \lambda]$, then the principal does not propose a contract. If $r \in (\lambda, \overline{r}_2)$, then the principal offers $(w^*, p^*) = \left(\overline{\eta}\overline{p}_{cu}/2 + 2\sqrt{(1-\overline{\eta})\overline{p}_{cu}\lambda} - \lambda, \overline{p}_{cu}\right)$ and the agent installs capacity $\mu^*(w^*, p^*) = \sqrt{(1-\overline{\eta})\overline{p}_{cu}\lambda} - \lambda$. If $r \in [\overline{r}_2, \overline{p}_2]$, then the principal offers $(w^*, p^*) = (\overline{w}_2, \overline{p}_2)$ and the agent installs capacity $\mu^*(w^*, p^*) = \sqrt{(1-\overline{\eta})\overline{p}_2\lambda} - \lambda$. If $r > \overline{p}_2$, then the principal offers $(w^*, p^*) = \left(2\sqrt{r\lambda} - \lambda, r\right)$ and the agent installs capacity $\mu^*(w^*, p^*) = \sqrt{r\lambda} - \lambda$. □

Theorem 5.17 indicates that the existence of a beneficial contract for weakly risk-seeking agent is determined exogenously by r, λ, and $\overline{\eta}$.

5.2 Optimal Strategies for the Moderately Risk-Seeking Agent

For the moderately risk-seeking agent we first derive the agent's optimal strategy. The agent's optimization problem is defined in (5.3).

Notation:

$$\overline{p}_3 \equiv \frac{2\lambda}{2 + \overline{\eta} - 2\sqrt{2\overline{\eta}}} \tag{5.23}$$

and the following identity is verified using the definition of $\overline{p}_3$:

$$\overline{w}_3 \equiv \left(1 - \frac{\overline{\eta}}{2}\right)\overline{p}_3 = 2\sqrt{\overline{p}_3\lambda} - \lambda \tag{5.24}$$

Note that $\overline{p}_3$ and $\overline{w}_3$ are functions of λ and $\overline{\eta}$. However we suppress the parameters $(\lambda, \overline{\eta})$.

Lemma 5.18. *Let* $2 > \overline{\eta} \geq 8/9$ *and* $\lambda > 0$*, then* $2\lambda/\left(2 + \overline{\eta} - 2\sqrt{2\overline{\eta}}\right) > 4\lambda$ *(see proof in the Appendix).*

We describe a moderately risk-seeking agent's optimal response to any $(w, p) \in \mathbb{R}^2_+$ in Proposition 5.19.

Proposition 5.19. *Consider a moderately risk-seeking agent* ($\overline{\eta} \in [8/9, 2)$).

(a) *Given*

$$p \in (0, \overline{p}_3) \text{ and } w \geq \left(1 - \frac{\overline{\eta}}{2}\right) p \tag{5.25}$$

then the agent accepts the contract and installs $\mu^*(w,p) = 0$ *with resulting expected utility rate* $u_A(\mu^*(w,p); w,p) = w - (1 - \overline{\eta}/2)p \geq 0$. *The agent rejects the contract if* $p \in (0, \overline{p}_3]$ *and* $w \in (0, (1 - \overline{\eta}/2)p)$.

(b) *Given*

$$p = \overline{p}_3 \text{ and } w \geq \overline{w}_3 \tag{5.26}$$

then the agent accepts the contract and is indifferent installing either $\mu^*(w,p) = 0$ *or* $\mu^*(w,p) = \sqrt{\overline{p}_3\lambda} - \lambda$. *In both cases the agent's expected utility rate is* $u_A(\mu^*(w,p); w,p) = w - (1 - \overline{\eta}/2)\overline{p}_3 = w - 2\sqrt{\overline{p}_3\lambda} + \lambda \geq 0$. *If* $r \in (0, \overline{p}_3]$*, then neither* $\mu^* = 0$ *nor* $\mu^* = \sqrt{\overline{p}_3\lambda} - \lambda$ *leads to admissible solutions (see Definition 5.1). If* $r > \overline{p}_3$*, then there exists* w^* *such that* $\left((w^*, \overline{p}_3), \mu^* = \sqrt{\overline{p}_3\lambda} - \lambda\right)$ *is the only admissible solution (for proof see Proposition 5.20). He rejects the contract if* $p = \overline{p}_3$ *and* $w \in (0, \overline{w}_3)$.

(c) *Given*

$$p > \overline{p}_3 \text{ and } w \geq 2\sqrt{p\lambda} - \lambda \tag{5.27}$$

then the agent accepts the contract and installs $\mu^*(w,p) = \sqrt{p\lambda} - \lambda$ *with resulting expected utility rate* $u_A(\mu^*(w,p); w,p) = w - 2\sqrt{p\lambda} + \lambda \geq 0$. *The agent rejects the contract if* $p > \overline{p}_3$ *and* $w \in \left(0, 2\sqrt{p\lambda} - \lambda\right)$.

Proof. According to Table 5.1, the optimization of $u(\mu)$ when $\overline{\eta} \in [8/9, 1)$ versus $\overline{\eta} \in [1, 2)$ is different. Therefore we prove the proposition separately for $\overline{\eta} \in [8/9, 1)$ and $\overline{\eta} \in [1, 2)$.

Case $\overline{\eta} \in [8/9, 1)$: According to Lemmas 5.6 and 5.18, $4\overline{p}_1 > \overline{p}_1 \geq \overline{p}_3 > 4\lambda$. Figure 5.8 shows the shape of $u(\mu)$ when $\overline{\eta} \in [8/9, 1)$ and the value of p falls in different ranges. The structure of the proof when $\overline{\eta} \in [8/9, 1)$ is depicted in Fig. 5.9.

Case $p \in (0, 4\lambda]$: According to Table 5.1, $u(\mu)$ is decreasing with respect to $\mu \geq 0$. Therefore the agent's optimal service capacity is $\mu^*(w, p) = 0$ and from Eq. (5.3) $u(\mu^*(w, p)) = w - (1 - \overline{\eta}/2)p$. Note that $1 - \overline{\eta}/2 > 0$.

Case $w \in \left(0, (1 - \overline{\eta}/2)p\right)$: $u(\mu^*(w, p)) < 0$, therefore the agent rejects the contract.

Case $w \geq (1 - \overline{\eta}/2)p$: $u(\mu^*(w, p)) \geq 0$, thus the agent would accept the contract if offered.

Case $p \in \left(4\lambda, \overline{p}_1\right]$: According to Table 5.1, there is a service capacity that maximizes $u(\mu)$ for $\mu \in [0, \lambda)$ and a service capacity that maximizes $u(\mu)$ for $\mu > \lambda$. Denote the optimal service capacity in $[0, \lambda)$ by $\mu^*_{[0,\lambda)}(w, p)$. Since $u(\mu)$ is decreasing with respect to μ over $[0, \lambda)$, therefore $\mu^*_{[0,\lambda)}(w, p) = 0$ and from (5.3) $u\left(\mu^*_{[0,\lambda)}(w, p)\right) = w - (1 - \overline{\eta}/2)p$. Denote the optimal service capacity for $\mu > \lambda$ by $\mu^*_{\lambda}(w, p)$. From first order condition $\mu^*_{\lambda}(w, p) = \sqrt{p\lambda} - \lambda$ and from (5.3) $u\left(\mu^*_{\lambda}(w, p)\right) = w - 2\sqrt{p\lambda} + \lambda$. The agent has to choose one of the two service capacities and he installs the one with higher expected utility rate. Note that $u\left(\mu^*_{\lambda}(w, p)\right) - u\left(\mu^*_{[0,\lambda)}(w, p)\right) = (1 - \overline{\eta}/2)p - 2\sqrt{p\lambda} + \lambda$. According to Lemma 5.5, $4\lambda > 2\lambda/\left(2 + \overline{\eta} + 2\sqrt{2\overline{\eta}}\right)$. According to Lemmas 5.6 and 5.18, $\overline{p}_1 \geq \overline{p}_3 > 4\lambda$, therefore we examine the following subcases.

Subcase $p \in \left(4\lambda, \overline{p}_3\right)$: By Lemma 5.4 part (a), $u\left(\mu^*_{[0,\lambda)}(w, p)\right) > u\left(\mu^*_{\lambda}(w, p)\right)$, therefore the agent's optimal service capacity is $\mu^*(w, p) = 0$ and $u(\mu^*(w, p)) = w - (1 - \overline{\eta}/2)p$. Note that $1 - \overline{\eta}/2 > 0$.

Subsubcase $w \in \left(0, (1 - \overline{\eta}/2)p\right)$: $u(\mu^*(w, p)) < 0$, therefore the agent rejects the contract.

Subsubcase $w \geq (1 - \overline{\eta}/2)p$: $u(\mu^*(w, p)) \geq 0$, thus the agent would accept the contract if offered.

Subcase $p = \overline{p}_3$: According to Lemma 5.4 part (c), $u\left(\mu^*_{[0,\lambda)}(w, p)\right) = u\left(\mu^*_{\lambda}(w, p)\right)$, indicating that installing $\mu^*_{[0,\lambda)}(w, \overline{p}_3)$ or $\mu^*_{\lambda}(w, \overline{p}_3)$ leads to the same agent's expected utility rate. Therefore the agent is indifferent about installing $\mu^*(w, p) = 0$ or $\mu^*(w, p) = \sqrt{p\lambda} - \lambda$. Still, the capacity value has to lead to admissible solutions (see Proposition 5.20). Recall that by definition $\overline{w}_3 = (1 - \overline{\eta}/2)\overline{p}_3$ (see (5.24)). Note that $1 - \overline{\eta}/2 > 0$.

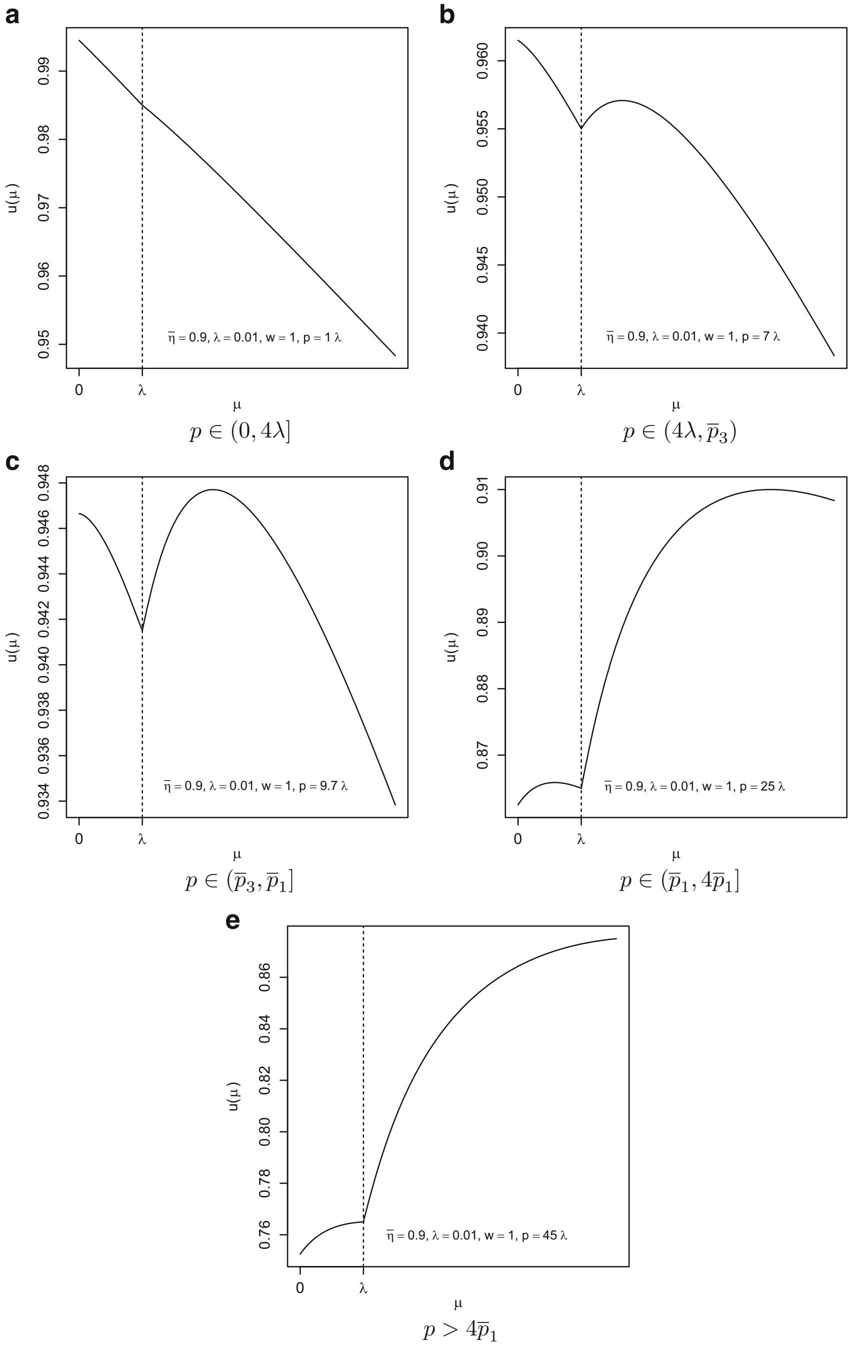


Fig. 5.8 Illustration of the forms of $u(\mu)$ when $\overline{\eta} \in [8/9, 1)$

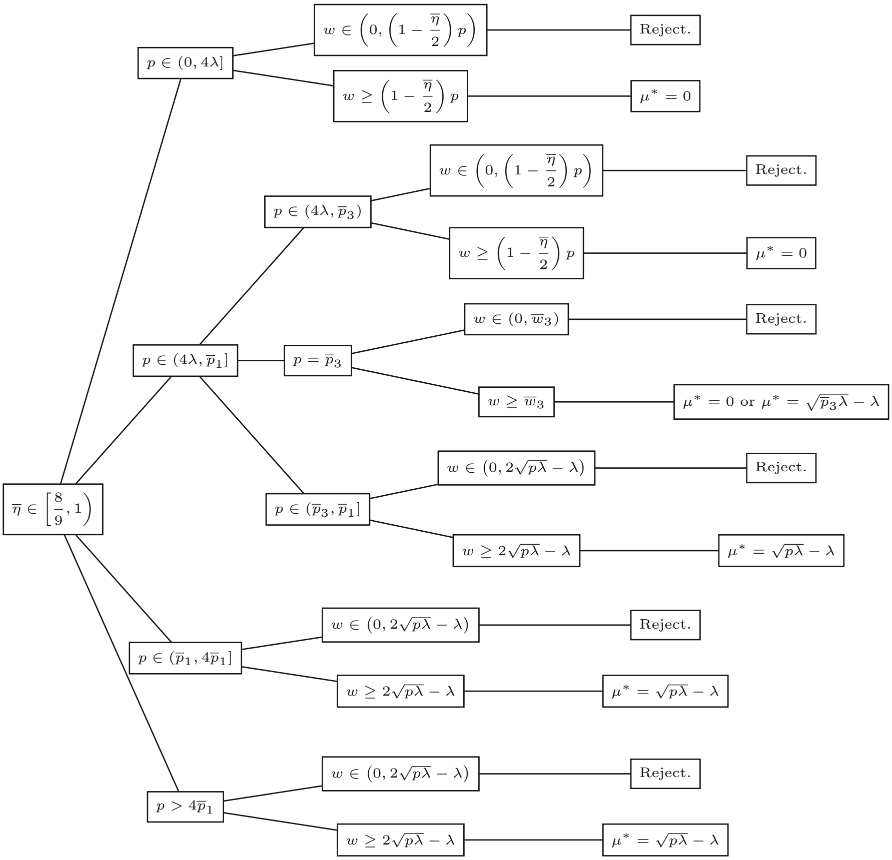


Fig. 5.9 Structure of the proof for Proposition 5.19 when $\overline{\eta} \in [8/9, 1)$

Subsubcase $w \in \left(0, (1 - \overline{\eta}/2)\overline{p}_3\right)$: $u(\mu^*(w, p)) < 0$, therefore the agent rejects the contract.

Subsubcase $w \geq (1 - \overline{\eta}/2)\overline{p}_3$: $u(\mu^*(w, p)) \geq 0$, therefore the agent would accept the contract if offered.

Subcase $p \in (\overline{p}_3, \overline{p}_1]$: According to Lemma 5.4 part (b), $u\left(\mu_\lambda^*(w, p)\right) > u\left(\mu_{[0,\lambda)}^*(w, p)\right)$, therefore the agent's optimal service capacity is $\mu^*(w, p) = \sqrt{p\lambda} - \lambda$ and $u(\mu^*(w, p)) = w - 2\sqrt{p\lambda} + \lambda$. Since $p > \overline{p}_3 > 4\lambda \Rightarrow 2\sqrt{p\lambda} - \lambda > 3\lambda > 0$, therefore we examine the following subcases.

Subsubcase $w \in \left(0, 2\sqrt{p\lambda} - \lambda\right)$: $u(\mu^*(w, p)) < 0$, thus the agent rejects the contract.

Subsubcase $w \geq 2\sqrt{p\lambda} - \lambda$: $u(\mu^*(w, p)) \geq 0$, therefore the agent would accept the contract if offered.

Case $p \in (\overline{p}_1, 4\overline{p}_1]$: According to Table 5.1, there is a service capacity that maximizes $u(\mu)$ for $\mu \in (0, \lambda]$ and a service capacity that maximizes $u(\mu)$ for $\mu > \lambda$. Denote the optimal service capacity in $(0, \lambda]$ by $\mu^*_{(0,\lambda]}(w, p)$. From first order condition the optimal service capacity is $\mu^*_{(0,\lambda]}(w, p) = \sqrt{(1-\overline{\eta})p\lambda} - \lambda$ and from (5.3) $u\left(\mu^*_{(0,\lambda]}(w, p)\right) = w - \overline{\eta}p/2 - 2\sqrt{(1-\overline{\eta})p\lambda} + \lambda$. Denote the optimal service capacity for $\mu > \lambda$ by $\mu^*_{\lambda}(w, p)$. From first order condition $\mu^*_{\lambda}(w, p) = \sqrt{p\lambda} - \lambda$ and from (5.3) $u\left(\mu^*_{\lambda}(w, p)\right) = w - 2\sqrt{p\lambda} + \lambda$. The agent has a choice of two service capacities and he installs the one that generates a higher expected utility rate. Note that $u\left(\mu^*_{\lambda}(w, p)\right) - u\left(\mu^*_{(0,\lambda]}(w, p)\right) = \overline{\eta}p/2 - 2\left(1 - \sqrt{1-\overline{\eta}}\right)\sqrt{p\lambda}$. According to Lemma 5.8, $\overline{p}_1 \geq \overline{p}_2$, therefore according to Lemma 5.7 part (b), $u\left(\mu^*_{\lambda}(w, p)\right) > u\left(\mu^*_{(0,\lambda]}(w, p)\right)$, the agent's optimal service capacity is $\mu^*(w, p) = \sqrt{p\lambda} - \lambda$ and $u(\mu^*(w, p)) = w - 2\sqrt{p\lambda} + \lambda$. Since $p > \overline{p}_1 > 4\lambda \Rightarrow 2\sqrt{p\lambda} - \lambda > 3\lambda > 0$, therefore we examine the following subcases.

Subcase $w \in \left(0, 2\sqrt{p\lambda} - \lambda\right)$: $u(\mu^*(w, p)) < 0$, therefore the agent rejects the contract.

Subcase $w \geq 2\sqrt{p\lambda} - \lambda$: $u(\mu^*(w, p)) \geq 0$, therefore the agent would accept the contract if offered.

Case $p > 4\overline{p}_1$: According to Table 5.1, the service capacity that maximizes $u(\mu)$ satisfies $\mu > \lambda$. From the first order condition the agent's optimal service capacity is $\mu^*(w, p) = \sqrt{p\lambda} - \lambda$ and $u(\mu^*(w, p)) = w - 2\sqrt{p\lambda} + \lambda$. Since $p > 4\overline{p}_1 > 4\lambda \Rightarrow 2\sqrt{p\lambda} - \lambda > 3\lambda > 0$, therefore we examine the following subcases.

Subcase $w \in \left(0, 2\sqrt{p\lambda} - \lambda\right)$: $u(\mu^*(w, p)) < 0$, thus the agent rejects the contract.

Subcase $w \geq 2\sqrt{p\lambda} - \lambda$: $u(\mu^*(w, p)) \geq 0$, therefore the agent would accept the contract if offered.

This complete the proof for Proposition 5.19 when $\overline{\eta} \in [8/9, 1)$.

Case $\overline{\eta} \in [1, 2)$: Note that $4\lambda > 0 > \overline{p}_1 > 4\overline{p}_1$ and according to Lemma 5.18, $\overline{p}_3 > 4\lambda$. Therefore $\overline{p}_3 > 4\lambda > 0 > \overline{p}_1 > 4\overline{p}_1$. Figure 5.10 depicts the shape of $u(\mu)$ when $\overline{\eta} \in [1, 2)$ and the value of p falls in different ranges. The structure of the proof when $\overline{\eta} \in [1, 2)$ is depicted in Fig. 5.11.

Case $p \in (0, 4\lambda]$: According to Table 5.1, $u(\mu)$ is decreasing with respect to $\mu \geq 0$. Therefore the agent's optimal service capacity is $\mu^*(w, p) = 0$ and from Eq. (5.3) $u(\mu^*(w, p)) = w - (1 - \overline{\eta}/2)p$. Note that $1 - \overline{\eta}/2 > 0$.

Subcase $w \in (0, (1 - \overline{\eta}/2)p)$: $u(\mu^*(w, p)) < 0$, therefore the agent rejects the contract.

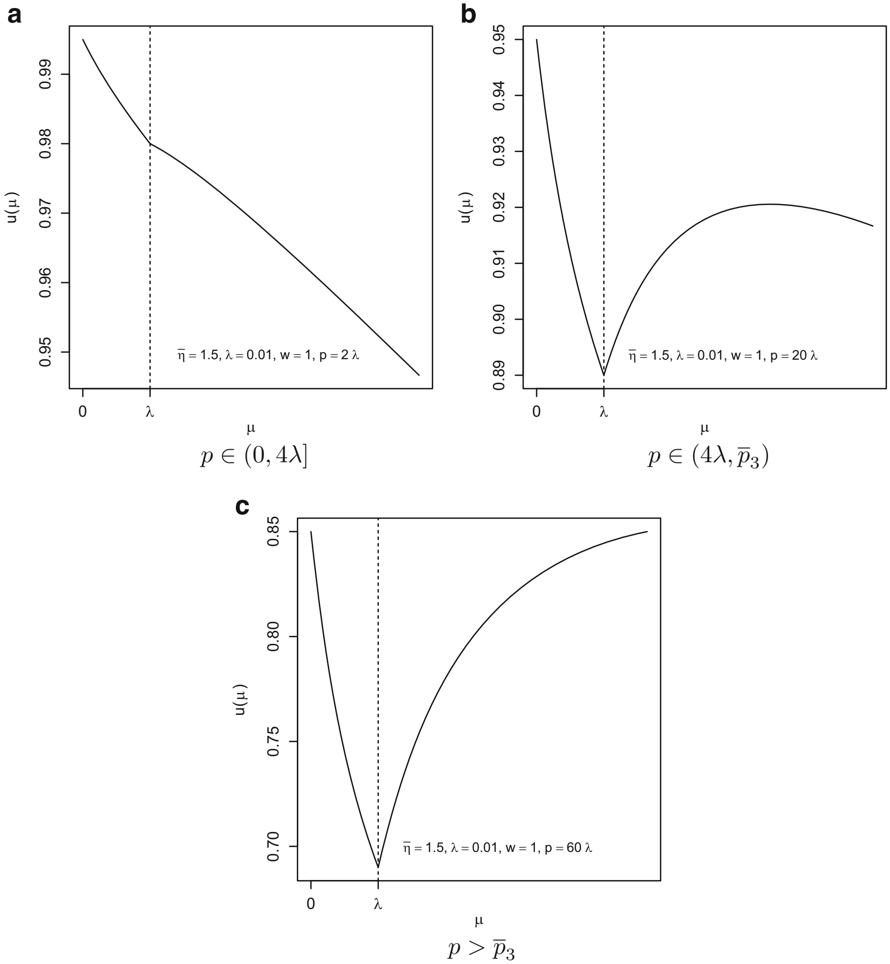


Fig. 5.10 Illustration of the forms of $u(\mu)$ when $\overline{\eta} \in [1, 2)$

Subcase $w \geq (1 - \overline{\eta}/2)p$: $u(\mu^*(w,p)) \geq 0$, thus the agent would accept the contract if offered.

Case $p > 4\lambda$: According to Table 5.1, there is a service capacity that maximizes $u(\mu)$ for $\mu \in [0, \lambda)$ and a service capacity that maximizes $u(\mu)$ for $\mu > \lambda$. Denote the optimal service capacity in $[0, \lambda)$ by $\mu^*_{[0,\lambda)}(w,p)$. Since $u(\mu)$ is decreasing with respect to μ over $[0, \lambda)$, therefore $\mu^*_{[0,\lambda)}(w,p) = 0$ and from (5.3) $u\left(\mu^*_{[0,\lambda)}(w,p)\right) = w - (1 - \overline{\eta}/2)p$. Denote the optimal service capacity for $\mu > \lambda$ by $\mu^*_\lambda(w,p)$. From first order condition $\mu^*_\lambda(w,p) = \sqrt{p\lambda} - \lambda$ and from (5.3) $u\left(\mu^*_\lambda(w,p)\right) = w - 2\sqrt{p\lambda} + \lambda$. The agent has to choose one of the

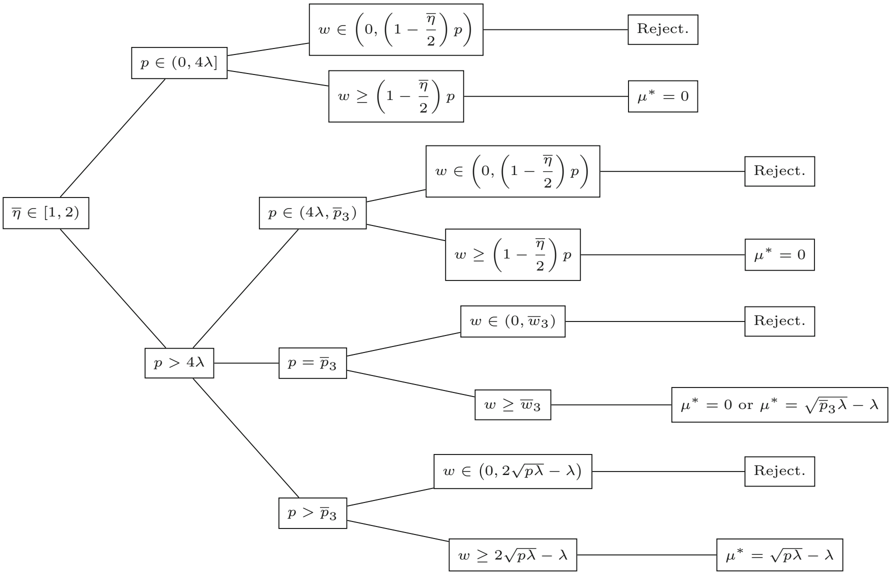


Fig. 5.11 Structure of the proof for Proposition 5.19 when $\overline{\eta} \in [1, 2)$

two service capacities and he installs the one with higher expected utility rate. Note that $u\left(\mu^*_\lambda(w,p)\right) - u\left(\mu^*_{[0,\lambda)}(w,p)\right) = (1-\overline{\eta}/2)p - 2\sqrt{p\lambda} + \lambda$. According to Lemma 5.5, $4\lambda > 2\lambda/\left(2+\overline{\eta}+2\sqrt{2\overline{\eta}}\right)$ and according to Lemma 5.18, $\overline{p}_3 > 4\lambda$, therefore we examine the following subcases.

Subcase $p \in (4\lambda, \overline{p}_3)$: By Lemma 5.4 part (a), $u\left(\mu^*_{[0,\lambda)}(w,p)\right) > u\left(\mu^*_\lambda(w,p)\right)$, therefore the agent's optimal service capacity is $\mu^*(w,p) = 0$ and $u(\mu^*(w,p)) = w - (1-\overline{\eta}/2)p$. Note that $1 - \overline{\eta}/2 > 0$.

Subsubcase $w \in \left(0, (1-\overline{\eta}/2)p\right)$: $u(\mu^*(w,p)) < 0$, therefore the agent rejects the contract.

Subsubcase $w \geq (1-\overline{\eta}/2)p$: $u(\mu^*(w,p)) \geq 0$, thus the agent would accept the contract if offered.

Subcase $p = \overline{p}_3$: According to Lemma 5.4 part (c), $u\left(\mu^*_{[0,\lambda)}(w,p)\right) = u\left(\mu^*_\lambda(w,p)\right)$, indicating that installing $\mu^*_{[0,\lambda)}(w,\overline{p}_3)$ or $\mu^*_\lambda(w,\overline{p}_3)$ leads to the same agent's expected utility rate. Therefore the agent is indifferent about installing $\mu^*(w,p) = 0$ or $\mu^*(w,p) = \sqrt{p\lambda} - \lambda$. Still, the capacity value has to lead to admissible solutions (see Proposition 5.20). Recall that by definition of $\overline{w}_3 = (1-\overline{\eta}/2)\overline{p}_3$ (see (5.24)) and $1 - \overline{\eta}/2 > 0 \Rightarrow \overline{w}_3 > 0$.

Subsubcase $w \in (0, \overline{w}_3)$: $u(\mu^*(w,p)) < 0$, therefore the agent rejects the contract.

Subsubcase $w \geq \overline{w}_3$: $u(\mu^*(w,p)) \geq 0$, thus the agent would accept the contract if offered.

Subcase $p > \overline{p}_3$: From Lemma 5.4 part (b), $u\left(\mu^*_\lambda(w,p)\right) > u\left(\mu^*_{[0,\lambda)}(w,p)\right)$, therefore the agent's optimal service capacity is $\mu^*(w,p) = \sqrt{p\lambda} - \lambda$ and $u(\mu^*(w,p)) = w - 2\sqrt{p\lambda} + \lambda$. Since $p > \overline{p}_3 > 4\lambda \Rightarrow 2\sqrt{p\lambda} - \lambda > 3\lambda > 0$, therefore we examine the following subcases.

Subsubcase $w \in \left(0, 2\sqrt{p\lambda} - \lambda\right)$: $u(\mu^*(w,p)) < 0$, thus the agent rejects the contract.

Subsubcase $w \geq 2\sqrt{p\lambda} - \lambda$: $u(\mu^*(w,p)) \geq 0$, therefore the agent would accept the contract if offered.

This completes the proof for Proposition 5.19 when $\overline{\eta} \in [1, 2)$. □

In summary, under the exogenous market conditions such that a contract between the principal and a moderately risk-seeking agent is feasible (see Theorem 5.22 later), only one formula is needed for the agent to compute his optimal service capacity: $\mu^*(w,p) = \sqrt{p\lambda} - \lambda > 0$.

The conditions when a moderately risk-seeking agent accepts the contract can be depicted by the shaded areas in Fig. 5.12, where $\overline{\eta} = 1$. The two shaded areas with different grey scales represent conditions (5.25) and (5.27) under which the agent accepts the contract but responds differently. The lower bound function of the shaded area (denoted by $w_0(p)$) represents the set of offers that give the agent zero expected utility rate. $w_0(p)$ is defined as follows:

$$w_0(p) = \begin{cases} \left(1 - \dfrac{\overline{\eta}}{2}\right) p \text{ when } p \in (0, \overline{p}_3] \\ 2\sqrt{p\lambda} - \lambda \text{ when } p > \overline{p}_3 \end{cases}$$

Since $\lim_{p \to \overline{p}_3^-} w_0(p) = \lim_{p \to \overline{p}_3^+} w_0(p) = \left(\sqrt{2} + \sqrt{\overline{\eta}}\right) \lambda / \left(\sqrt{2} - \sqrt{\overline{\eta}}\right)$, therefore $w_0(p)$ is continuous everywhere over interval $p \in \mathbb{R}_+$. However since $\lim_{p \to \overline{p}_3^-} dw_0(p)/dp = 1 - \overline{\eta}/2 \neq 1 - \sqrt{\overline{\eta}/2} = \lim_{p \to \overline{p}_3^+} dw_0(p)/dp$, therefore $w_0(p)$ is not differentiable at $p = \overline{p}_3$.

5.2.1 Sensitivity Analysis of a Moderately Risk-Seeking Agent's Optimal Strategy

Since the principal does not propose a contract that even if accepted will result in zero service capacity, therefore the only viable case is when the agent accepts the contract and installs positive service capacity: $\mu^*(w,p) = \sqrt{p\lambda} - \lambda$. In such a case the agent's optimal strategy is identical to the optimal strategy for risk-neutral agent. According to (5.10) the compensation rate w is bounded below by $2\sqrt{p\lambda} - \lambda = pP(1) + \mu^*(w,p)$, with the term $pP(1)$ representing the expected penalty rate charged by the principal when the optimal capacity is installed. It indicates that the

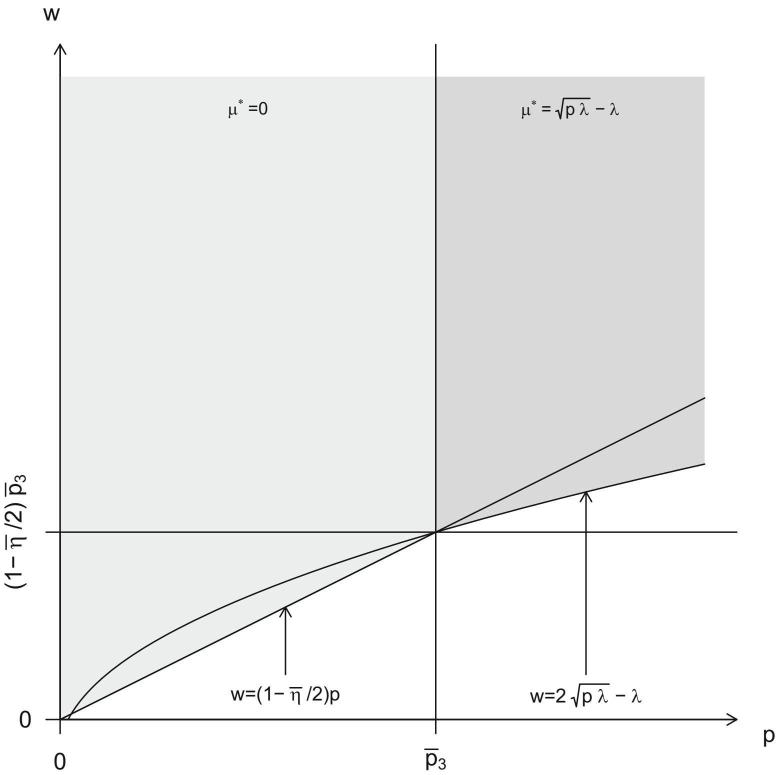


Fig. 5.12 Conditions when a moderately risk-seeking agent accepts the contract with $\overline{\eta} = 1$

agent has to be reimbursed for the expected penalty rate and the cost of service capacity.

The optimal service capacity $\sqrt{p\lambda} - \lambda$ depends on the penalty rate p and the failure rate λ. Its derivatives are $\partial\mu^*/\partial p = \sqrt{\lambda/4p} > 0$ and $\partial\mu^*/\partial\lambda = \sqrt{p/4\lambda} - 1$. These derivatives suggest that given the failure rate, the agent will increase the service capacity when the penalty rate increases. Note that $\sqrt{p\lambda} - \lambda$, as a function of λ, increases when $p/4 > \lambda$. From conditions (5.26) and (5.27) the agent installs service capacity $\sqrt{p\lambda} - \lambda$ when $p \geq \overline{p}_3$, and according to Lemma 5.18 we have $\overline{p}_3 > 4\lambda$. Therefore we have $p > 4\lambda \Rightarrow p/4 > \lambda \Rightarrow \partial\mu^*/\partial\lambda > 0$. Thus, given the penalty rate, the agent will increase the service capacity when the failure rate increases.

The agent's optimal expected utility rate when installing capacity $\mu^*(w, p) = \sqrt{p\lambda} - \lambda$ is $u_A^* \equiv u_A(\mu^*(w, p); w, p) = w - 2\sqrt{p\lambda} + \lambda$, and it depends on w, p and λ only. Note that $\partial u_A^*/\partial w = -1 < 0$, $\partial u_A^*/\partial p = -\sqrt{\lambda/p} < 0$, indicating that the agent's optimal expected utility rate decreases with the compensation rate and the penalty rate. Note that $\partial u_A^*/\partial\lambda = -\sqrt{p/\lambda} + 1$, and from Proposition 5.10

$p \geq \overline{p}_3 > 4\lambda \Rightarrow -\sqrt{p/\lambda} + 1 < 0$, therefore the agent's optimal expected utility rate also decreases with the failure rate.

Summary: Recall that given the set of offers $\{(w,p) : p \in (0,\lambda], w \geq p\}$ a risk-neutral agent would accept the contract, install $\mu^*(w,p) = 0$. When $\left\{(w,p) : p > \lambda, w \geq 2\sqrt{p\lambda} - \lambda\right\}$ he would accept the contract, install $\mu^*(w,p) = \sqrt{p\lambda} - \lambda$ and realize an expected utility rate $u(\mu^*(w,p); w,p) = w - 2\sqrt{p\lambda} + \lambda$. By comparing the optimal capacities of a moderately risk-seeking agent to that of a risk-neutral agent, three conclusions are drawn.

1. The principal has to set a higher p in order to induce a moderately risk-seeking agent to install a positive service capacity versus a risk-neutral agent ($p > \lambda$ for risk-neutral agent, $p > \overline{p}_3 = 2\lambda/\left(\sqrt{2} - \sqrt{\overline{\eta}}\right)^2 > \lambda$ for moderately risk-seeking agent).
2. A moderately risk-seeking agent would install the same positive service capacity as a risk-neutral agent ($\sqrt{p\lambda} - \lambda$).
3. Given w and p, an agent who accepts the contract and subsequently installs a positive service capacity will receive the same expected utility rate when he is moderately risk-seeking ($\overline{\eta} \in [8/9, 2)$) as risk-neutral ($\overline{\eta} = 0$).

5.2.2 *Principal's Optimal Strategy*

We now proceed to derive the principal's optimal strategy. Anticipating the agent's optimal selection of $\mu^*(w,p)$ the principal chooses w and p to maximize her expected profit rate by solving the optimization problem

$$\max_{w>0,p>0} \Pi_P(w,p;\mu^*(w,p)) = \max_{w>0,p>0} \left\{ \frac{r\mu^*(w,p)}{\lambda + \mu^*(w,p)} - w + \frac{p\lambda}{\lambda + \mu^*(w,p)} \right\} \tag{5.28}$$

where the principal's optimal solution values are $(w^*, p^*) = \text{argmax}_{w>0,p>0} \Pi_P(w,p;\mu^*(w,p))$.

Before we describe the principal's optimal strategy, we reexamine the case when the principal offers $p = \overline{p}_3$ and $w \geq \overline{w}_3$, under which the agent is indifferent about installing two different service capacities. The selected solutions $((w,p),\mu)$ have to be admissible solutions (see Definition 5.1). We state this case formally in Proposition 5.20.

Proposition 5.20. *Suppose a moderately risk-seeking agent and principal. Assume that the principal's offers are constrained to* $\{(w,p) : p = \overline{p}_3, w \geq \overline{w}_3\}$.

(a) *If* $r \in (0, \overline{p}_3]$, *then the principal does not propose a contract.*
(b) *If* $r > \overline{p}_3$, *then the agent installs* $\mu^* = \sqrt{\overline{p}_3\lambda} - \lambda$ *if offered a contract.*

Proof. Note that for $w \geq \overline{w}_3$ we have $\partial \Pi_P(w, \overline{p}_3; \mu)/\partial \mu = (r - \overline{p}_3)\lambda/(\lambda + \mu)^2$. Define $\mu_L \equiv 0$ and $\mu_H \equiv \sqrt{\overline{p}_3 \lambda} - \lambda$ and note that $\mu_H > \mu_L$. If $r \in (0, \overline{p}_3)$, then $\partial \Pi_P/\partial \mu < 0$, therefore $((w, \overline{p}_3), \mu_L) \succeq ((w, \overline{p}_3), \mu_H)$ and the agent would install μ_L if offered a contract. However condition (c) in Definition 5.1 requires that $w \geq \overline{p}_3$, therefore $\Pi_P(w, \overline{p}_3; \mu_L) = -w + \overline{p}_3 \leq 0$ and the principal would not propose a contract. If $r = \overline{p}_3$, then $\partial \Pi_P/\partial \mu = 0$, therefore the agent installs either μ_L or μ_H if offered a contract. However in such case the principal's expected profit rate is $\Pi_P(w, \overline{p}_3; \mu_L) = \Pi_P(w, \overline{p}_3; \mu_H) = -w + \overline{p}_3$, which is non-positive due to condition (c) in Definition 5.1, thus the principal would not propose a contract. If $r > \overline{p}_3$, then $\partial \Pi_P/\partial \mu > 0$ and $((w, \overline{p}_3), \mu_H) \succeq ((w, \overline{p}_3), \mu_L)$. If the principal offers a contract (where the conditions will be discussed in detail in Theorem 5.22 that follows), then by Definition 5.1 only μ_H leads to admissible solutions. □

Lemma 5.21. *Consider* $\max_{x \geq \sqrt{\overline{p}_3}} f(x)$ *where* $f(x) = r + \lambda - \sqrt{\lambda}\,(x + r/x)$ *and denote* $x^* = \operatorname{argmax}_{x \geq \sqrt{\overline{p}_3}} f(x)$. *The solutions to this optimization problem are (see proof in the Appendix):*

(a) $x^* = \sqrt{\overline{p}_3}$ *if* $r \in (0, \overline{p}_3]$.
(b) $x^* = \sqrt{r}$ *if* $r > \overline{p}_3$.

The principal's optimal strategy is described in Theorem 5.22. Recall that Proposition 5.19 describes the agent's optimal response to each pair $(w, p) \in \mathbb{R}^2_+$. Since the principal will not propose a contract that is going to be rejected by a moderately risk-seeking (MRS) agent, therefore Theorem 5.22 only considers pairs $(w, p) \in \mathbb{R}^2_+$ that result in agent's non-negative expected utility rate. Define

$$\begin{aligned} \mathfrak{D}_{(5.25)} &\equiv \{(w,p) \text{ that satisfies (5.25) when } \overline{\eta} \in [8/9, 2)\} \\ \mathfrak{D}_{(5.26)} &\equiv \{(w,p) \text{ that satisfies (5.26) when } \overline{\eta} \in [8/9, 2)\} \\ \mathfrak{D}_{(5.27)} &\equiv \{(w,p) \text{ that satisfies (5.27) when } \overline{\eta} \in [8/9, 2)\} \\ \mathfrak{D}_{\text{MRS}} &\equiv \mathfrak{D}_{(5.25)} \cup \mathfrak{D}_{(5.26)} \cup \mathfrak{D}_{(5.27)} \end{aligned} \tag{5.29}$$

Theorem 5.22. *Given a moderately risk-seeking agent and* $(w, p) \in \mathfrak{D}_{\text{MRS}}$.

(a) *If* $r \in (0, \overline{p}_3]$, *then the principal does not propose a contract.*
(b) *If* $r > \overline{p}_3$, *then the principal's offer and the capacity installed by the agent are*

$$(w^*, p^*) = \left(2\sqrt{r\lambda} - \lambda, r\right) \text{ and } \mu^*(w^*, p^*) = \sqrt{r\lambda} - \lambda \tag{5.30}$$

and the principal's expected profit rate is

$$\Pi_P(w^*, p^*; \mu^*(w^*, p^*)) = r - 2\sqrt{r\lambda} + \lambda \tag{5.31}$$

Proof. The structure of the proof for Theorem 5.22 is depicted in Fig. 5.13.

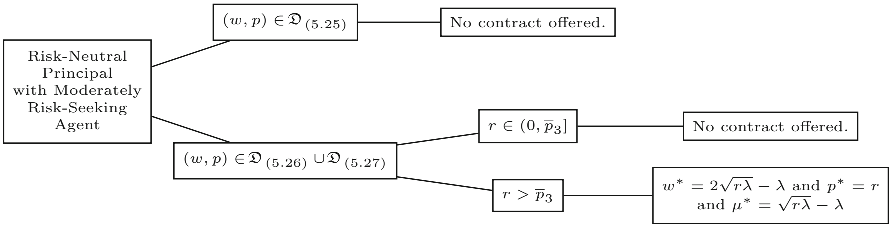


Fig. 5.13 Structure of the proof for Theorem 5.22

Case $(w,p) \in \mathfrak{D}_{(5.25)}$: According to Proposition 5.19 part (a), in case the principal makes an offer, the agent accepts the contract but does not install any service capacity. Since $\partial \Pi_P / \partial w = -1 < 0$, thus we have $w^* = (1 - \overline{\eta}/2)p$ and from (3.3) $\Pi_P(w^*, p; \mu^*(w^*, p)) = -w^* + p = \overline{\eta}p/2 > 0$. However in such case $p > w^* = (1 - \overline{\eta}/2)p$, which violates condition (c) in Definition 5.1, therefore $((w^* = (1 - \overline{\eta}/2)p, p), \mu^* = 0)$ is not an admissible solution and the principal does not propose a contract.

Case $(w,p) \in \mathfrak{D}_{(5.26)} \cup \mathfrak{D}_{(5.27)}$: According to Proposition 5.19 part (c), if $(w,p) \in \mathfrak{D}_{(5.27)}$, then in case the principal makes an offer, the agent accepts the contract and installs $\sqrt{p\lambda} - \lambda$. Since $\partial \Pi_P / \partial w = -1 < 0$, therefore $w^* = 2\sqrt{p\lambda} - \lambda$. According to Propositions 5.19 part (b) and 5.20, if $(w,p) \in \mathfrak{D}_{(5.26)}$ (which implies $p = \overline{p}_3$), then the principal does not propose a contract if $r \in (0, \overline{p}_3]$, or installs $\sqrt{\overline{p}_3\lambda} - \lambda$ in case the principal makes an offer when $r > \overline{p}_3$. Since $\partial \Pi_P / \partial w = -1 < 0$, therefore $w^* = \overline{w}_3$. Denote the principal's expected profit rate when $(w,p) = (\overline{w}_3, \overline{p}_3)$ and $\mu^*(w,p) = 0$ by $\Pi_P^L(\overline{p}_3)$, and denote the principal's expected profit rate when $(w,p) = (\overline{w}_3, \overline{p}_3)$ and $\mu^*(w,p) = \sqrt{\overline{p}_3\lambda} - \lambda$ by $\Pi_P^H(\overline{p}_3)$. By plugging the value of w, p and μ into (3.3):

$$\Pi_P^L(\overline{p}_3) = -\left(1 - \frac{\overline{\eta}}{2}\right)\overline{p}_3 + \overline{p}_3 = \frac{\overline{\eta}\overline{p}_3}{2} \tag{5.32}$$

$$\Pi_P^H(\overline{p}_3) = r + \lambda - \sqrt{\lambda}\left(\sqrt{\overline{p}_3} + \frac{r}{\sqrt{\overline{p}_3}}\right) \tag{5.33}$$

In such case the principal's optimization problem is $\max_{p \geq \overline{p}_3} \Pi_P(w^*, p; \mu^*(w^*, p))$ where:

$$\Pi_P(w^*, p; \mu^*(w^*, p)) = \begin{cases} \max\left\{\Pi_P^L(\overline{p}_3), \Pi_P^H(\overline{p}_3)\right\}, \text{for } p = \overline{p}_3 \\ r + \lambda - \sqrt{\lambda}\left(\sqrt{p} + \dfrac{r}{\sqrt{p}}\right), \text{for } p > \overline{p}_3 \end{cases}$$

Define $x \equiv \sqrt{p}$, the expression $r + \lambda - \sqrt{\lambda}\left(\sqrt{p} + r/\sqrt{p}\right)$ can be restated as $f(x) = r + \lambda - \sqrt{\lambda}\,(x + r/x)$. Maximizing $f(x)$ with respect to $x \geq \sqrt{\overline{p}_3}$ is equivalent to maximizing $r + \lambda - \sqrt{\lambda}\left(\sqrt{p} + r/\sqrt{p}\right)$ with respect to $p \geq \overline{p}_3$ in the sense that

$$\underset{p \geq \overline{p}_3}{\operatorname{argmax}} \left\{ r + \lambda - \sqrt{\lambda}\left(\sqrt{p} + \frac{r}{\sqrt{p}}\right)\right\} = \left(\underset{x \geq \sqrt{\overline{p}_3}}{\operatorname{argmax}} f(x)\right)^2$$

Therefore we examine the following subcases.

Subcase $r \in (0, \overline{p}_3]$: According to Lemma 5.21 part (a), $p^* = \overline{p}_3$ and according to Proposition 5.20 part (a) the principal does not propose a contract.

Subcase $r > \overline{p}_3$: According to Lemma 5.21 part (b), $p^* = r$ and the principal's expected profit rate is $\Pi_P(w^*, p^*; \mu^*(w^*, p^*)) = r - 2\sqrt{r\lambda} + \lambda > \Pi_P^H(\overline{p}_3) > \Pi_P^L(\overline{p}_3) = \overline{\eta}\overline{p}_3/2 > 0$. Thus the principal proposes a contract with $w^* = 2\sqrt{r\lambda} - \lambda$ and $p^* = r$ that induces the agent to install $\mu^*(w^*, p^*) = \sqrt{r\lambda} - \lambda$.

To summarize, if $r \in (0, \overline{p}_3]$, then the principal does not propose a contract. If $r > \overline{p}_3$, then the principal offers $(w^*, p^*) = \left(2\sqrt{r\lambda} - \lambda, r\right)$ and the agent installs capacity $\mu^*(w^*, p^*) = \sqrt{r\lambda} - \lambda$. □

Theorem 5.22 indicates that the existence of a contract for moderately risk-seeking agent is determined exogenously by the r, λ, and $\overline{\eta}$.

5.3 Optimal Strategies for the Strongly Risk-Seeking Agent

We start by deriving the strongly risk-seeking agent's optimal strategy. The agent's optimization problem is defined in (5.3).

First a technical lemma (see proof in the Appendix).

Lemma 5.23. *Let $\overline{\eta} > 2$ and $\lambda > 0$.*

(a) *If* $\dfrac{2\lambda}{2 + \overline{\eta} + 2\sqrt{2\overline{\eta}}} > p > 0$, *then* $\left(1 - \dfrac{\overline{\eta}}{2}\right)p - 2\sqrt{p\lambda} + \lambda > 0$.

(b) *If* $p > \dfrac{2\lambda}{2 + \overline{\eta} + 2\sqrt{2\overline{\eta}}}$, *then* $0 > \left(1 - \dfrac{\overline{\eta}}{2}\right)p - 2\sqrt{p\lambda} + \lambda$.

(c) *If* $p = \dfrac{2\lambda}{2 + \overline{\eta} + 2\sqrt{2\overline{\eta}}}$, *then* $\left(1 - \dfrac{\overline{\eta}}{2}\right)p - 2\sqrt{p\lambda} + \lambda = 0$.

We describe a strongly risk-seeking agent's optimal response to any possible offered contract $(w, p) \in \mathbb{R}_+^2$ in Proposition 5.24.

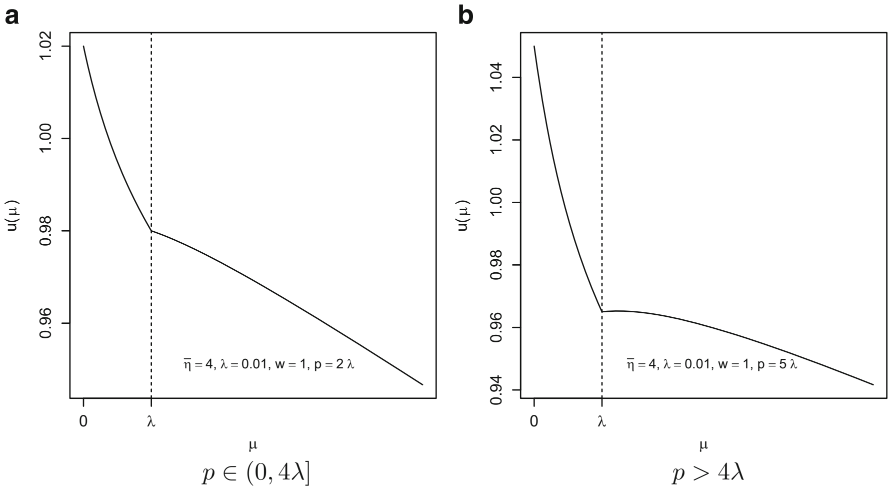
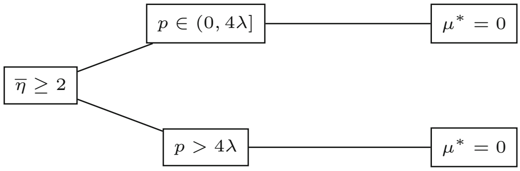


Fig. 5.14 Illustration of the forms of $u(\mu)$ when $\overline{\eta} \geq 2$

Fig. 5.15 Structure of the proof for Proposition 5.24 when $\overline{\eta} \geq 2$



Proposition 5.24. *Consider a strongly risk-seeking agent* ($\overline{\eta} \geq 2$). $\forall\ w > 0$ *and* $p > 0$, *the agent accepts the contract and installs* $\mu^*(w,p) = 0$ *with resulting expected utility rate* $u_A(\mu^*(w,p); w,p) = w - (1 - \overline{\eta}/2)p > 0$.

Proof. Figure 5.14 shows the shape of $u(\mu)$ when $\overline{\eta} \geq 2$ and the value of p falls in different ranges. The structure of the proof when $\overline{\eta} \geq 2$ is depicted in Fig. 5.15.

Case $p \in (0, 4\lambda]$: According to Table 5.1, $u(\mu)$ is decreasing with respect to $\mu \geq 0$. Therefore the agent's optimal service capacity is $\mu^*(w,p) = 0$ and from (5.3) $u(\mu^*(w,p)) = w - (1 - \overline{\eta}/2)p$. Note that $1 - \overline{\eta}/2 \leq 0$, therefore $\forall\ w > 0$, $u(\mu^*(w,p)) > 0$ and the agent would accept the contract if offered.

Case $p > 4\lambda$: According to Table 5.1, there is a service capacity that maximizes $u(\mu)$ for $\mu \in [0, \lambda)$ and a service capacity that maximizes $u(\mu)$ for $\mu > \lambda$. Denote the optimal service capacity in $[0, \lambda)$ by $\mu^*_{[0,\lambda)}(w,p)$. Since $u(\mu)$ is decreasing with respect to μ over $[0, \lambda)$, therefore $\mu^*_{[0,\lambda)}(w,p) = 0$ and from (5.3) $u\left(\mu^*_{[0,\lambda)}(w,p)\right) = w - (1 - \overline{\eta}/2)p$. Denote the optimal service capacity for $\mu > \lambda$ by $\mu^*_\lambda(w,p)$. From first order condition $\mu^*_\lambda(w,p) = \sqrt{p\lambda} - \lambda$ and from (5.3) $u\left(\mu^*_\lambda(w,p)\right) = w - 2\sqrt{p\lambda} + \lambda$. The agent has to choose one of the two service capacities and he installs the one with higher expected utility rate. Note that $u\left(\mu_\lambda^*(w,p)\right) - u\left(\mu_{[0,\lambda)}^*(w,p)\right) = (1-\overline{\eta}/2)p - 2\sqrt{p\lambda} + \lambda$. If $\overline{\eta} = 2$, then $u\left(\mu_\lambda^*(w,p)\right) - u\left(\mu_{[0,\lambda)}^*(w,p)\right) = -2\sqrt{p\lambda} + \lambda$, and since $p > 4\lambda \Leftrightarrow 2\sqrt{p\lambda} > 4\lambda \Leftrightarrow 0 > -3\lambda > -2\sqrt{p\lambda}+\lambda$, we have $u\left(\mu_{[0,\lambda)}^*(w,p)\right) > u\left(\mu_\lambda^*(w,p)\right)$. If $\overline{\eta} > 2$, then according to Lemmas 5.5 and 5.23 part (b), $p > 4\lambda > 2\lambda/\left(2+\overline{\eta}+2\sqrt{2\overline{\eta}}\right) \Rightarrow u\left(\mu_{[0,\lambda)}^*(w,p)\right) > u\left(\mu_\lambda^*(w,p)\right)$. Thus the agent's optimal service capacity is $\mu^*(w,p) = 0$ and $u(\mu^*(w,p)) = w - (1-\overline{\eta}/2)p$. Note that $1-\overline{\eta}/2 \leq 0$, therefore $\forall\ w > 0$, $u(\mu^*(w,p)) > 0$ and the agent would accept the contract if offered. □

Proposition 5.24 indicates that a strongly risk-seeking agent does not commit any capacity, therefore the principal does not propose any contract, which we state formally in Theorem 5.25.

Theorem 5.25. *A principal never offers a contract to a strongly risk-seeking agent.*

Proof. According to Proposition 5.24, the agent accepts the contract but does not install any service capacity for all $(w,p) \in \mathbb{R}_+^2$. In such case the principal's expected profit rate is $\Pi_P(w,p;\mu^*(w,p)) = -w + p$. Since condition (c) of Definition 5.1 requires that $w \geq p$, therefore $\Pi_P(w,p;\mu^*(w,p)) \leq 0$ and the principal does not propose a contract to a strongly risk-seeking agent! □

5.4 Risk-Seeking Agent: A Summary

Recall the definition of $\overline{p}_2$, $\overline{r}_2$ and $\overline{p}_3$ from (5.5), (5.12) and (5.23). The conditions when a principal makes contract offers to a risk-seeking agent is depicted by the shaded areas in Fig. 5.16. The horizontal axis represents the agent's risk coefficient $\overline{\eta}$, and the vertical axis represents the revenue rate generated by the principal's equipment unit, which is exogenously determined by the market. The principal makes different offers to the agent when $(r,\overline{\eta})$ is in the three shaded areas with different gray scales. We define

$$\overline{p}_{23} \equiv \begin{cases} \overline{p}_2 \text{ for } \overline{\eta} \in (0, 8/9) \\ \overline{p}_3 \text{ for } \overline{\eta} \in [8/9, 2) \end{cases}$$

Since $\lim_{\overline{\eta}\to(8/9)^-} \overline{p}_{23} = \lim_{\overline{\eta}\to(8/9)^+} \overline{p}_{23} = 9\lambda$ and $\lim_{\overline{\eta}\to(8/9)^-} \partial\overline{p}_{23}/\partial\overline{\eta} = \lim_{\overline{\eta}\to(8/9)^+} \partial\overline{p}_{23}/\partial\overline{\eta} = 81\lambda/4$, and note that $\lim_{\overline{\eta}\to2^-} \overline{p}_{23} = \lim_{\overline{\eta}\to2^-} \overline{p}_3 = +\infty$, therefore $\overline{p}_{23}$ is continuous and differentiable everywhere over $(0,2)$. In Fig. 5.16 we only describe the conditions of a risk-neutral principal making offers to a weakly and moderately risk-seeking agent ($\overline{\eta} \in (0,2)$), because the principal never makes a contract offer to a strongly risk-seeking agent ($\overline{\eta} \geq 2$).

The revenue rate parameter r is determined exogenously by the market, and we assume that the principal is only interested in operating the equipment when the

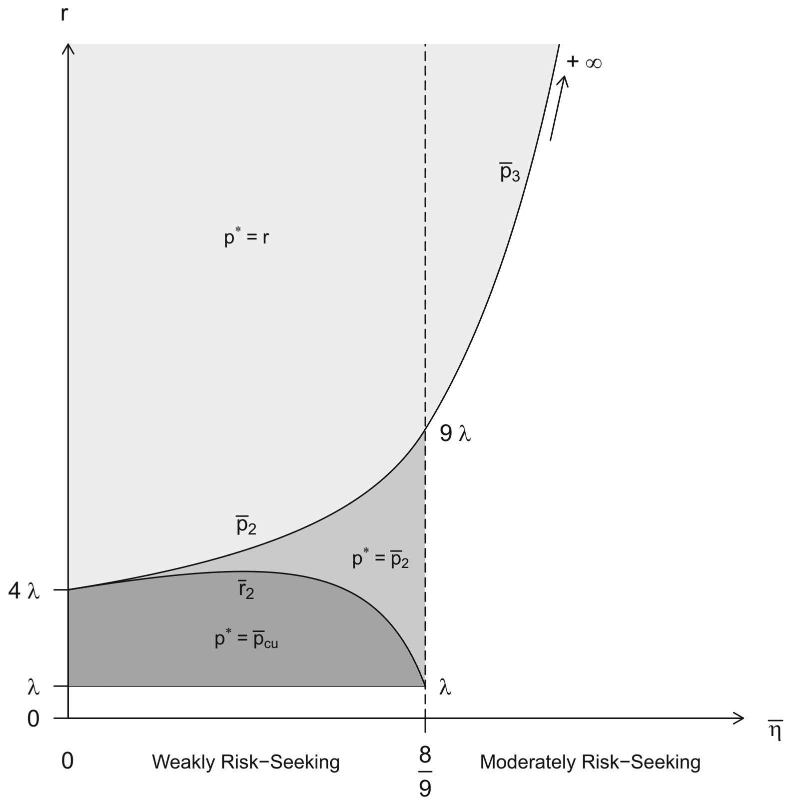


Fig. 5.16 Conditions when a principal makes contract offers to a risk-seeking agent

revenue rate is sufficiently high, specifically $r > \overline{p}_{23}$. In such case weakly and moderately risk-seeking agents would behave exactly the same as a risk-neutral agent, and a strongly risk-seeking agent will never be offered a contract.

Chapter 6
Summary

In this paper we examine a basic principal-agent arrangement for contracting an exclusive equipment repair service supplier. The system setting consists of one principal, one agent, and one revenue generating unit that breaks down from time to time and needs to be repaired when a failure occurs. Our assumptions are that the risk-neutral principal maximizes her expected profit rate given market driven revenue rate r collected during the unit's uptime, the unit's failure rate λ, and the agent's risk attitude η. We consider different agent types – risk neutral, weakly risk-averse, strongly risk-averse, weakly risk-seeking, moderate risk-seeking, and strongly risk-seeking. As is common in a principal-agent context the principal cannot contract directly for the agent's service capacity μ. The nature of the principal-agent contract is that the principal supports the agent at a compensation rate $w > 0$ but imposes on the agent a penalty rate $p > 0$ during the time the unit is down. We note that the nature of the contract does not change if the w is paid to the agent only during the unit's uptime. In fact, the two contract versions are equivalent (see Observation 3.1).

The main contribution of this paper is in the complete analysis of the contractual details that have to be addressed in the agreement between the unit's owner and the supplier of repair services. Our pedestrian assumptions are that the failure rate of the equipment unit is a constant λ, the repair time duration has an exponential distribution with a constant repair rate μ. Furthermore, we do not restrict the contract to a specific period of time, rather the contract can be for undetermined time. With the assumption that both the principal and the agent are infinitely rational the surprising outcome is that calculating the optimal strategies for the two parties in all circumstances can be accomplished with an aid of small number of formulas – 7 sets in total. That is, given exogenously determined values of market driven revenue rate, equipment's failure rate, repair capacity marginal cost, and the type of a repair agent, it is straight forward to calculate principal's optimal contract offer if one exists, together with agent's optimal service capacity decision. An optimal

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contract consists of compensation rate w and penalty rate p, both determined by the principal, and the capacity value of μ determined by the agent.

Our analysis of the above principal-agent cooperation is divided into three main parts based on agent's type starting with risk-neutral agent. The second part examines the case of a contracting a risk-averse agent followed by the analysis of a contract given a risk-seeking agent. To our knowledge analysis of principal-agent with risk-seeking agent has not received much coverage in the literature.

As for the analysis of principal-agent construct given a risk-neutral agent, for the entire range of exogenous parameters' values, it can be summarized for the principal by one set of formulas calculating optimal compensation rate $w^* = 2\sqrt{r\lambda} - \lambda$ and optimal penalty rate $p^* = r$. The agent's optimal capacity rate formula is $\mu^*(w^*, p^*) = \sqrt{r\lambda} - \lambda$. We note that this case has the property that without checking if the given market conditions guarantee the existence of a contract, by calculating principal's optimal contract terms w^* and p^* and agent's optimal capacity value $\mu^*(w^*, p^*)$, we simultaneously verify contract existence if the resulting $\mu^*(w^*, p^*)$ is positive. If the optimal capacity value is zero or negative, then it means that the given market conditions do not support a service contract. It also important to note that, for our principal-agent given a risk-neutral agent, if an optimal contract is feasible then it is also efficient.

When considering a risk-averse agent the first task is to decide on the appropriate mathematical expression that captures the agent's disutility with regard to his revenue dispersion. After examining risk premium expressions in the literature we opted for a new risk expression not yet seen in the literature. We express agent's disutility as $\eta p(1/2 - |1/2 - \lambda/(\lambda + \mu)|)$. This measure of agent's utility value due to his revenue fluctuation is introduced and discussed in Chap. 4. The main points are that the risk expression acts like standard deviation and is unit-wise compatible with other terms of agent's utility. In high revenue industry, if the principal contracts with a risk-averse agent with the risk disutility measured by the dispersion of the agent's revenue stream, then agent's risk-aversion reduces the principal's optimal penalty rate and leads to deterioration of the equipment unit's performance. Furthermore, with risk-averse agent the principal is strictly worse off in relation to risk-neutral agent and the social welfare is reduced as the agent's risk-aversion increases.

We divided risk-averse agents into two types based on risk intensity parameter η. That is, for $\eta \in (0, 4/5)$ we refer to the agent as *weakly risk-averse* (Sect. 4.1) and for $\eta \geq 4/5$ we refer to the agent as *strongly risk-averse* (Sect. 4.2). A weakly risk-averse agent has only two formulas to consider: (i) $\mu^*(w, p) = \sqrt{(1-\eta)p\lambda} - \lambda$ or (ii) $\mu^*(w, p) = \sqrt{(1+\eta)p\lambda} - \lambda$. Only one formula, the same as (ii), is sufficient given a strongly risk-averse agent. Formula (i) exists only for WRA agent because when the penalty rate is low, the savings from reducing the service capacity is more prominent than the increase in the penalty charge, providing an incentive for the agent to reduce the optimal service capacity, which deteriorates the performance of the principal's equipment unit. When the penalty rate becomes high, WRA agent increases his service capacity to reduce the penalty charge, which results in formula (ii).

For a risk-seeking agent we adopt a risk premium expression that reflects the expected amount at stake instead of the dispersion of his revenue stream. Our new risk premium expression is consistent with the theoretical developments and empirical evidences regarding the properties of risk in recent literature. We express agent's risk premium as $-\eta p\,(\lambda/(\lambda+\mu)-1/2)_+$, which is unit-wise compatible with other terms of agent's utility (see Chap. 5). If the principal contracts with a risk-seeking agent with low penalty rate, then the agent's risk-seeking deteriorates the performance of the principal's equipment unit. If the principal contract with a risk-seeking agent with high penalty rate, then she can achieve the same equipment performance and contract efficiency as with a risk-neutral agent. However a principal never contracts with a strongly risk-seeking agent.

We categorize risk-seeking agents into three types based on $\overline{\eta}$ – risk intensity parameter. That is, for $\overline{\eta} \in (0, 8/9)$ we refer to the agent as *weakly risk-seeking* (Sect. 5.1), for $\overline{\eta} \in [8/9, 2)$ we refer to the agent as *moderately risk-seeking* (Sect. 5.2) and the agent as *strongly risk-seeking* (Sect. 5.3). A weakly risk-seeking agent has only two formulas to consider: (i) $\mu^*(w,p) = \sqrt{(1-\overline{\eta})p\lambda} - \lambda$ or (ii) $\mu^*(w,p) = \sqrt{p\lambda} - \lambda$. Only one formula, the same as (ii), is sufficient given a moderately risk-seeking agent. A strongly risk-seeking agent never commits any service capacity. Formula (i) exists only for WRS agent because when the penalty rate is low, the risk premium covers the penalty charge thus provides an incentive for the agent to reduce the optimal service capacity compared to risk-neutral. When the penalty rate increases, WRS agent increases his service capacity to reduce the penalty charge that cannot be covered by risk premium, which results in formula (ii).

6.1 Interpreting Table 6.1

Table 6.1 summarizes the formulas for calculating the principal's optimal contract terms and the agent's optimal service capacity when a contract is supported by exogenous market and industry conditions. Mutually exclusive exogenous conditions that support a contract are listed in the column labeled "Exogenous Condition", and the formulas of the principal's optimal contract terms and the agent's optimal capacities are listed in the column labeled "Principal's Formula" and "Agent's Formula" respectively.

If a set of specific market and industry values are observed, namely the value of the agent's risk coefficient η (or $\overline{\eta}$), the revenue rate r, and the failure rate λ, then these values can be validated against the exogenous conditions listed in the table. If the set of values satisfies a certain condition, then the principal's formula and the agent's formula corresponding to that condition can be used to calculate the optimal contract terms and the optimal capacity. No contract is supported if the set of values does not satisfy any condition listed in the table.

To verify that whether the observed values of η, r, and λ satisfy a certain condition, one has to calculate the values that separate the range of r into different

Table 6.1 Summary of the optimal principal-agent contract formulas under exogenous conditions

Exogenous Condition			
Agent's Type	Revenue	Principal's Formula	Agent's Formula
$\eta = 0$ (RN)	$r > \lambda$	$(w^*, p^*) = \left(2\sqrt{r\lambda} - \lambda, r\right)$	$\mu^*(w^*, p^*) = \sqrt{r\lambda} - \lambda$
$\eta \in \left(0, \frac{4}{5}\right)$ (WRA)	$r \in (p_2, p_3]$	$(w^*, p^*) = \left(\eta p_{cu} + 2\sqrt{(1-\eta)p_{cu}\lambda} - \lambda, p_{cu}\right)$	$\mu^*(w^*, p^*) = \sqrt{(1-\eta)p_{cu}\lambda} - \lambda$
	$r \in (p_3, r_2)$	$(w^*, p^*) = \left(\eta p_{cu} + 2\sqrt{(1-\eta)p_{cu}\lambda} - \lambda, p_{cu}\right)$	$\mu^*(w^*, p^*) = \sqrt{(1-\eta)p_{cu}\lambda} - \lambda$
		$(w^*, p^*) = (w_3, p_3)$	$\mu^*(w^*, p^*) = \sqrt{(1+\eta)p_3\lambda} - \lambda$
	$r \in [r_2, r_3]$	$(w^*, p^*) = (w_3, p_3)$	$\mu^*(w^*, p^*) = \sqrt{(1+\eta)p_3\lambda} - \lambda$
	$r > r_3$	$(w^*, p^*) = \left(2\sqrt{\frac{(1+\eta)r\lambda}{1+2\eta}} - \lambda, \frac{r}{1+2\eta}\right)$	$\mu^*(w^*, p^*) = \sqrt{\frac{(1+\eta)r\lambda}{1+2\eta}} - \lambda$
$\eta \geq \frac{4}{5}$ (SRA)	$r \in (p_4, r_4]$	$(w^*, p^*) = (w_4, p_4)$	$\mu^*(w^*, p^*) = \sqrt{(1+\eta)p_4\lambda} - \lambda$
	$r > r_4$	$(w^*, p^*) = \left(2\sqrt{\frac{(1+\eta)r\lambda}{1+2\eta}} - \lambda, \frac{r}{1+2\eta}\right)$	$\mu^*(w^*, p^*) = \sqrt{\frac{(1+\eta)r\lambda}{1+2\eta}} - \lambda$
$\overline{\eta} \in \left(0, \frac{8}{9}\right)$ (WRS)	$r \in (\lambda, \overline{r}_2)$	$(w^*, p^*) = \left(\frac{\overline{\eta}\overline{p}_{cu}}{2} + 2\sqrt{(1-\overline{\eta})\overline{p}_{cu}\lambda} - \lambda, \overline{p}_{cu}\right)$	$\mu^*(w^*, p^*) = \sqrt{(1-\overline{\eta})\overline{p}_{cu}\lambda} - \lambda$
	$r \in [\overline{r}_2, \overline{p}_2]$	$(w^*, p^*) = (\overline{w}_2, \overline{p}_2)$	$\mu^*(w^*, p^*) = \sqrt{(1-\overline{\eta})\overline{p}_2\lambda} - \lambda$
	$r > \overline{p}_2$	$(w^*, p^*) = \left(2\sqrt{r\lambda} - \lambda, r\right)$	$\mu^*(w^*, p^*) = \sqrt{r\lambda} - \lambda$
$\overline{\eta} \in \left[\frac{8}{9}, 2\right)$ (MRS)	$r > \overline{p}_3$	$(w^*, p^*) = \left(2\sqrt{r\lambda} - \lambda, r\right)$	$\mu^*(w^*, p^*) = \sqrt{r\lambda} - \lambda$

intervals, including p_2, p_3, r_2, r_3, p_4, r_4, $\overline{r}_2$, $\overline{p}_2$, and $\overline{p}_3$. Recall that p_2 and p_3 are defined in (4.5), r_2 and r_3 are defined in (4.12), p_4 and w_4 are defined in (4.23), r_4 is defined in (4.28), $\overline{p}_2$ is defined in (5.5), $\overline{r}_2$ is defined in (5.12), and $\overline{p}_3$ is defined in (5.23). Furthermore, to calculate the principal's optimal contract terms and the agent's optimal capacity, one may need to calculate the values of p_{cu}, w_3, $\overline{p}_{cu}$, and $\overline{w}_2$. Recall that p_{cu} can be calculated using (4.13), w_3 is defined in (4.6), $\overline{p}_{cu}$ can be calculated using (5.13), and $\overline{w}_2$ is defined in (5.6).

Specifically, note that when the revenue rate $r \in (p_3, r_2)$, there are two sets of formulas listed in the table to calculate the principal's optimal contract terms and the agent's optimal capacity:

$$(w^*, p^*) = \left(\eta p_{cu} + 2\sqrt{(1-\eta)p_{cu}\lambda} - \lambda, p_{cu}\right), \mu^*(w^*, p^*) = \sqrt{(1-\eta)p_{cu}\lambda} - \lambda$$

$$(w^*, p^*) = (w_3, p_3), \mu^*(w^*, p^*) = \sqrt{(1+\eta)p_3\lambda} - \lambda$$

According to Proposition 4.20 it is difficult to identify the principal's optimal offer when $r \in (p_3, r_2)$ due to the difficulty of computing p_{cu} (see Eq. (4.13)). However, given the value of η, r, and λ, the principal's expected profit rate of both offers can be calculated (see the formulas for calculating the principal's expected profit rate in Proposition 4.20), and the offer with higher expected profit rate should be selected by the principal.

In summary, this paper provides a small set of formulas that exhaustively covers the computing of Pareto optimal principal-agent contract offer and corresponding service capacity for any values of market and industry parameters.